Information-Based Pricing in Specialized Lending *

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Abstract

We study specialized lending in a credit market competition model with private information. Two banks, equipped with similar data processing systems, possess general signals regarding the borrowers quality; the specialized bank, has access to an additional specialized signal. We study equilibria in which both lenders use general signals to screen loan applications. Conditional on making an offer, the specialized lender prices loans based on its specialized signal. This private-information-based pricing helps explain why loans made by specialized lenders have lower interest rates (lower winning bids) and better ex-post performance (fewer non-performing loans), which we support with robust empirical evidence.

JEL Classification: G21, L13, L52, O33, O36

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1 Introduction

It has long been recognized by the literature (e.g., Broecker, 1990; Riordan, 1993; Hauswald and Marquez, 2003) that competition among informed financial intermediaries in the credit market is central to the efficiency of financial systems. Of significant importance, banks hold a diverse array of lending-related information, including financial data on customers, collateral evaluations, and market and economic trends, not to mention state-of-the-art data analytics. Moreover, as shown in Blickle, Parlatore, and Saunders (2023), certain banks may accumulate specialized knowledge by concentrating on lending to particular industries, through the acquisition and analysis of diverse information on the business practices of individual firms and industries.

Despite the remarkable technological advancement that could significantly impact the industrial landscape of the banking sector, the prevailing literature (Marquez, 2002; Hauswald and Marquez, 2003; He, Huang, and Zhou, 2023) on information-based credit market competition predominantly focuses on binary signal realizations, neglecting the complexities of the advanced practices mentioned above. To this goal, we study credit market competition with specialized lending, where one (specialized) lender with general and specialized signals competes against another (non-specialized) lender with a general signal only. Importantly, the specialized lender's extra continuous signal is crucial in setting its equilibrium fine-tuned loan pricing. This novel multi-dimensional information setting, incorporated into an otherwise classic credit market competition model (a la Broecker, 1990), allows us to study private-information-based pricing in specialized lending.

Taking as a starting point the finding in (Blickle, Parlatore, and Saunders, 2023) that banks specialize their lending to certain industries, we motivate our model with a simple empirical exercise. Using regulatory loan-level data from the Y14-Q Schedule H database maintained by the Fed, for each year in our sample, we compute the difference between the average interest rate of loans granted by specialized banks in their industry of specialization and those of their loans in other industries. Figure 1 shows these measures since 2012. There, we see specialized lenders consistently charge around 40 basis points less for loans in their specialized industry. This difference is on "winning bids" rather than "bids"—as our loan-level data is based on granted loans, not loan offers—which is an important distinction through the lens of our credit market equilibrium model. Equally important, Figure 1 shows that specialized lenders are less likely to encounter non-performing loans in their industry of specialization. The empirical regularity documented in Figure 1, which is robust to more stringent econometric specifications and potential competition among specialized banks (as shown in Section 4.2), suggests that specialized lenders can identify better borrowers and "undercut" the non-specialized opponent lenders in their specialized industries.

The existing information-based models, e.g., Broecker (1990) and Marquez (2002), fail to deliver the above empirical regularity. There, each lender has a binary signal and actively competes only upon receiving a positive signal realization, offering interest rates drawn from a completely randomized mixed strategy. Hence, in these canonical models, the interest rate per se carries no information. As Section 4.1 shows, a stark information rent effect dominates in that canonical setting, under which the loans on the book of a stronger lender (with a more precise signal) tend

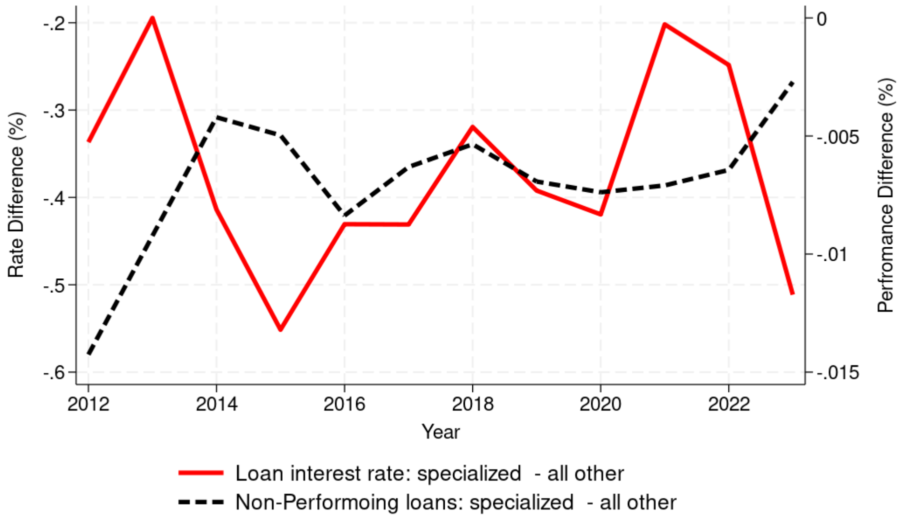

Figure 1: **Differences in interest rates and loan performance between specialized and non-specialized lenders.** We define specialized lenders as those with more than 4% over-investment in an industry, where over-investment is measured as deviations from a diversified portfolio $\frac{LoanAmount_{b,i,t}}{\sum_s LoanAmount_{b,i,t}} - \frac{LoanAmount_{i,t}}{\sum_i LoanAmount_{i,t}}$ for bank b in industry i at time t. The red solid line (left-hand side scale) plots the average difference between loan annual interest rates in the bank's specialized industry and those outside of its specialized industry. The dashed black line (right-hand side scale) plots the average annual differences in the fraction of non-performing loans when comparing loans in a bank's specialized industry against its other loans. The patterns are robust to various specifications of specialized lenders and volume-based weights; for details, see Appendix B. For a more in-depth discussion of measures of bank specialization, see Blickle, Parlatore, and Saunders (2023).

to have higher interest rates. This prediction is counterfactual in light of Figure 1.

In our model, a specialized bank competes with a non-specialized bank. Each lender has a "general" information signal on the loan quality from data processing. Moreover, the specialized lender has access to an additional signal from "specialized" information about the borrower, based on which the lender decides an interest rate to offer. We further assume that, while the general signal is binary and decisive in that each lender makes an offer only upon receiving a positive general signal, the specialized signal—which differentiates our paper from existing models—is continuous and guides the fine-tuned interest rate offering of the specialized bank.[1]

As highlighted in Section 2.2, our main analysis focuses on a multiplicative structure (similar to the O-ring theory) so that project success requires two distinct fundamental states "general" and "specialized" to be favorable;[2] and the aforementioned two types of signals—general and

[1]Besides providing analytical convenience, this loan-making rule matches the lending practices observed in practice. Essentially, in our model, the specialized bank acquires two signals, one being "principal" while the other being "supplementary;" the former determines whether to lend while the latter affects the detailed pricing terms. The principal signal can also represent the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

[2]This setting is quite general, as the general and specialized fundamental states can potentially overlap. To the extreme, these two fundamental states coincide entirely, and our model becomes the standard setting where one single fundamental state dictates the overall quality of the project.

specialized—inform the lenders regarding these two states, respectively. In Section 3, we fully characterize the competitive credit market equilibrium with specialized lending in closed form. In equilibrium, the specialized bank's interest rate schedule is *decreasing* in its specialized signal. Since the successful project's payoff is capped, our specialized bank—even conditional on a positive general signal—withdraws from the competition after receiving a sufficiently unfavorable specialized signal. In contrast, the non-specialized bank behaves just like in Broecker (1990) fully randomizing its interest rate offers. Therefore by incorporating both general and specialized signals, our model delivers the key result of private-information-based pricing.[3]

We derive a unique credit market equilibrium, which can fall into two distinct categories depending on the competitiveness of the banking industry. In the first category of equilibria, the winner's curse dominates, pushing the non-specialized "weak" bank to earn zero profits; we call it a zero-weak equilibrium. In this case, the non-specialized bank randomly withdraws when receiving a positive general signal, consequently yielding more monopoly power to its specialized opponent. In the second category of equilibria, the winner's curse is less severe and the non-specialized bank makes a positive profit in equilibrium (therefore always participates upon a positive general signal); we call it a positive-weak equilibrium.

We discuss the model's implications in Section 4. We focus on the empirical regularity that loans of specialized lenders have lower rates, which we call the "negative interest rate wedge." First, we highlight the difference between *bids* (i.e., offered interest rates) and *winning bids* (offered rates accepted by the borrower). This distinction is crucial when loan rejections are an important part of equilibrium strategies, as is typical in credit competition models. Although the standard winner's curse effect pushes the weaker lender to quote higher interest rates, in credit market competition models like He, Huang, and Zhou (2023) the weak lender also responds by rejecting loan applications. In equilibrium the strong lender exerts its monopolistic power by quoting the maximum interest rate randomly (which might be accepted in equilibrium), resulting in a higher expected rate for granted loans by specialized lenders. We call this the *information rent* effect.

In contrast, by modeling specialized signals, we explicitly incorporate the specialized lender's "undercutting" to win creditworthy borrowers, favoring a lower expected rate for granted loans by specialized lenders. We call this the "private-information-based pricing" effect. We highlight that this effect prevails especially in the positive-weak regime: there the specialized bank has less monopoly power and hence makes more aggressive offers to get good borrowers, as explained above.[4]

[3] Conceptually, this is similar to the common value auction setting in Milgrom and Weber (1982), where the informed buyer who privately observes a continuum of signal realizations bids monotonically based on its private information (see literature review for more details). In addition, one could extend the range of quoted interest rates by borrowers to include infinity and interpret $r = \infty$ as "rejection;" this way the lenders in the classic credit market competition model in Broecker (1990) also have private-information-based pricing. However, Figure 1 is constructed based on interest rates of granted loans, which excludes $r = \infty$; we stress this point in Section 4.1 where we discuss the distinction between "bids" and "winning bids."

[4] Consistent with information-based pricing, Butler (2008) finds local investment banks charge lower fees and issue municipal bonds at lower yields than non-local underwriters. On the other hand, Degryse and Ongena (2005) finds that local banks charge higher interest rates to small firms, consistent with local banks' strong monopolistic power over hard-to-evaluate captive borrowers.

As one of the main results of our paper, Section 4.1 shows that canonical credit competition models cannot generate the empirical regularity of a "negative interest rate wedge." We show that under empirically relevant parameters, the information rent effect dominates in canonical credit competition models a la Broecker (1990), yielding the counterfactual implications that loans by specialized lenders have higher rates. In contrast, our private-information-based pricing effect helps deliver a "negative interest rate wedge." Our mechanism differs from those of Mahoney and Weyl (2017) and Crawford, Pavanini, and Schivardi (2018). As we explain toward the end of Section 4.1, in that literature market power (of lenders) and adverse selection (of borrowers) are treated as two distinct market frictions, whereas our model features the winner's curse as the only underlying force for both market power and adverse selection.

We explore several extensions. First, we show our equilibrium characterization is robust to a generalized information structure that allows for correlated general and specialized signals. The key to our analytical tractability is the multiplicative structure and its resulting "independence conditional on success," i.e., all signals, the two general ones and the specialized one, are independent conditional on project success. Second, we endogenize the information structure by considering two ex-ante symmetric banks competing in two industries. Lenders can invest in a general information technology (fixed cost, binary signal of borrower quality) and also acquire costly, firm-specific specialized information (continuous signal) to become specialized; each lender only needs to invest once in the general information technology for the two industries but has to acquire the specialized signal separately for each industry. We provide conditions for a "symmetric" specialization equilibrium, where each industry has one specialized and one non-specialized lender, as in our baseline model.

The remainder of the paper is organized as follows. After a brief literature review, Section 2 presents the baseline model. Section 3 characterizes the credit market equilibrium and Section 4 explores the economic implications of our model, with several extensions. Section 5 concludes.

Literature Review

Lending market competition and common-value auctions. Our paper builds on Broecker (1990), which studies lending market competition with screening tests and symmetric lenders (i.e., with the same screening abilities). Relatedly, Hauswald and Marquez (2003) explores the competition between an inside bank that can conduct credit screenings and an outside bank without such access, and He, Huang, and Zhou (2023) consider competition between asymmetric lenders with different screening abilities under open banking when borrowers control access to data.[5] In these models, for analytical tractability, it is often assumed that private screening yields a binary signal and lenders participate only when receiving the positive signal realization. In contrast to these papers, we consider competition between asymmetrically informed lenders with multiple information sources.

Theoretically, credit market competition models are an application of common-value auctions.

[5]Asymmetric credit market competition can also naturally arise from the bank-customer relationship, as a bank knows its existing customers better than a new competitor. This idea was explored by a two-period model in Sharpe (1990) where asymmetric competition arises in the second period (with the corrected analysis of a mixed-strategy equilibrium offered by Von Thadden (2004)). A similar analysis is present in Rajan (1992).

Notably, the auction literature typically allows for general signal distributions (other than the binary signal in the aforementioned papers).[6] For instance, Riordan (1993) extends the N-symmetric-lender model in Broecker (1990) to a setting with continuous private signals. In terms of modeling, our framework can be viewed as a combination of Broecker (1990) (symmetric bidders with general signals) and Milgrom and Weber (1982) (asymmetric bidders, one with a specialized signal). Our model analyzes credit market competition with specialized lenders, an application where asymmetric screening technologies are crucial. Also, in our model lenders are each privately informed with potentially different general signals. This breaks the Blackwell ordering of the information of two lenders in Milgrom and Weber (1982), resulting in a considerably more challenging problem.[7]

Specialization in lending. There is growing literature documenting specialization in bank lending; for an early paper, see Acharya, Hasan, and Saunders (2006). Paravisini, Rappoport, and Schnabl (2023) show that Peruvian banks specialize their lending across export markets benefiting borrowers who obtain credit from their specialized banks. Based on data for US stress-tested banks, Blickle, Parlatore, and Saunders (2023) shows that banks specialize their portfolios in different industries in a way consistent with them having larger informational advantages in industries in which they specialize more. This informational advantage manifests as better loan performance at the cost of some aggregate profitability in the industry in which the bank specializes relative to all other industries in the portfolio. Our paper contributes to this literature in two ways. First, we focus on the effects of competition among specialized and non-specialized lenders within an industry. We show that specialized banks have fewer non-performing loans issued at lower rates in their portfolios than non-specialized banks in the same industry, not due to competition among specialized banks. Second, we provide a framework that can rationalize observed specialization patterns, allowing us to better understand the economic mechanisms behind them and their implications.[8]

The connection to imperfect competition and adverse selection in the IO literature. The empirical pattern and our theoretical analyses on the negative interest rate wedge between asymmetrically informed lenders are connected to the industrial organization (IO) literature on imperfect competition and adverse selection (Mahoney and Weyl, 2017; Crawford, Pavanini, and Schivardi, 2018; Yannelis and Zhang, 2023). As we explain in detail in Section 4.1, different from the IO literature which takes market power (of lenders) and adverse selection (of borrowers) as two independent market frictions, our theory is based on "asymmetric information" which is a more primitive assumption, with winner's curse faced by asymmetrically informed lenders as the only underlying economic force. Strictly speaking, in our model, there is no "market power" enjoyed by the specialized lender as money from any funding source is perfectly fungible; and, there is no "adverse

[6]The early papers on this topic include Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom, and Weber (1983), and later papers such as Hausch (1987); Kagel and Levin (1999) explore information structures where each bidder has some private information, which is the information structure adopted in Broecker (1990).

[7]More precisely, one bidder knows strictly more than the other bidder. In this setting, one can show that the under-informed bidder always makes zero profit; see also Engelbrecht-Wiggans, Milgrom, and Weber (1983).

[8]Our paper also connects to the growing literature on fintech disruption; see Berg, Fuster, and Puri (2021); Vives (2019), for instance, for a review of fintech companies competing with traditional banks in originating loans.

selection" from borrowers either, as both types of borrowers will take loans at any interest rate.[9]

2 Model

In this section, We lay out the model and define the equilibrium accordingly.

2.1 General Setting

We consider a credit market competition model with two dates, one good, and risk-neutral agents (two lenders and one borrower). There are two lenders (banks) indexed by $j \in \{A, B\}$, where Bank A (B) is the specialized (non-specialized) lender.

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow y at $t = 1$. The cash flow realization y depends on the project's quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$y = \begin{cases} 1 + \overline{r}, & \text{when } \theta = 1, \\ 0, & \text{when } \theta = 0, \end{cases} \tag{1}$$

where $\overline{r} > 0$ is exogenously given, i.e., only a good project has a positive NPV. We will later refer to $\overline{r}$ as the interest rate cap or the return of a good project. The project's quality θ is unobservable to lenders, and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$.

Credit market competition. At date $t = 0$, each bank j can choose to make a take-it-or-leave-it interest rate offer $r^j \leq \overline{r}$ of a fixed loan amount of one to the borrower or to make no offer (i.e., exit the lending market), which we normalize as $r^j = \infty$. The borrower accepts the offer with the lowest rate if it receiving multiple offers.[10]

Information technology. Banks have access to information about the borrower's project quality before choosing whether to make an offer. We assume that both lenders have access to "general" data (say financial and operating data), which they can process to produce a *general-information-*based private signal g^j. We call these information "general" signals. We assume that these general signals are binary, i.e., $g^j \in \{H, L\}$, with a realization H (L) being a positive (negative) signal; and

[9] Our paper is also related to the literature on the nature of information in bank lending. Berger and Udell (2006) provides a comprehensive framework of the two fundamental types of bank lending technology, i.e., relationship lending and transactions lending, in the SME lending market; these two types of lending are related to the role played by information as highlighted by Stein (2002); Paravisini and Schoar (2016). Recently, based on Harte Hanks data, He, Jiang, Xu, and Yin (2023) show a significant rise in IT investment within the U.S. banking sector over the past decade, particularly among large banks, and their causal link between communication IT spending and the enhancement of banks' capacity in generating and transmitting soft information motivates our modeling of the specialized signal as the outcome of interactions with borrowers.

[10] We implicitly assume that borrowers obtain some (however small) private benefit, so it is strictly optimal to take the project even for the type $\theta = 0$. One important implication is that it is irrelevant whether borrowers privately know θ or not, as both types of borrowers always pool in equilibrium.

that, conditional on the (relevant) state, general signals are independent across lenders. Besides following the traditional structure presented in Broecker (1990), this modeling of general signals also captures the coarseness with which some general information is used in practice. For example, as a leading example of "general information," credit scores are binned in five ranges even though scores are computed at a much granular level and go from 300 to 850.

Additionally, we endow Bank A with a signal s, which captures the bank being "specialized." As the major departure from the existing literature, this additional signal is as a *specialized-information*-based private signal, which is collected, for example, after due diligence or face-to-face interactions with the borrower after on-site visits. We assume that the specialized signal s is continuous, and its distribution is characterized by the Cumulative Distribution Function (CDF) $\Phi(s)$ and probability density function (pdf) $\phi(s)$. Besides providing mathematical convenience, the continuous distribution captures "specialized" signals resulting from research tailored to the particular borrower and, therefore, allows for a more granular assessment of the borrower's quality.

The information structure is incomplete unless we specify the correlations between the fundamental states and the two types of signals, to which we turn in the next section.

2.2 The Setting with a Multiplicative Structure

General and specialized fundamental states. Our main analysis focuses on the specific setting with a multiplicative structure for the state θ, so that

$$\theta \equiv \theta_g \theta_s \equiv \begin{cases} 1, & \text{when } \theta_g = \theta_s = 1, \\ 0, & \text{when either } \theta_g = 0 \text{ or } \theta_s = 0. \end{cases} \tag{2}$$

Here, $\theta_g \in \{0, 1\}$ captures the "*g*eneral" state and $\theta_s \in \{0, 1\}$ captures the "*s*pecialized" state; they jointly determine the project's success θ, in that the project fails when *either* state fails.

We further assume that general and specialized states are independent, so that the prior probability of the state being "1" is simply $q = q_g q_s$ with $q_g \equiv \mathbb{P}(\theta_g = 1)$ and $q_s \equiv \mathbb{P}(\theta_s = 1)$. This independence, together with the independence of the noise across signals, implies complete independence between the generalized and specialized signals (for Bank A).

The distribution of the signals conditional on the state reflects the information technology. We assume that conditional on the state, the signal realizations are independent across borrowers. It is straightforward to allow for correlated signals conditional on the state (see He, Huang, and Parlatore (2024)). For general information signals, which are assumed to be binary, we assume

$$\mathbb{P}\left(g^j = H \,|\theta_g = 1\right) = \alpha_u \in [0,1]\,,\ \mathbb{P}\left(g^j = L \,|\theta_g = 0\right) = \alpha_d \in [0,1]\,, \quad \text{for} \quad j \in \{A, B\}\,. \tag{3}$$

Here, the information technology is not indexed by lender j—that is to say, lenders have the same technology to process general information that comes from "general" sources like financial statements, an assumption that we relax in Section 4.3.

In (3), $1-\alpha_u$ and $1-\alpha_d$ capture the probabilities of Type I and Type II errors, respectively. The bad-news signal structure in He, Huang, and Zhou (2023) corresponds to $\alpha_u = 1$ and a symmetric signal structure has $\alpha_u = \alpha_d = \alpha \in (0.5, 1]$ as in Hauswald and Marquez (2003) and He, Jiang, and Xu (2024). Our main numerical illustration focuses on the latter case, although our solution is robust to any $\{\alpha_u, \alpha_d\}$ structure.

For the continuous specialized signal, without loss of generality, we directly work with the posterior of the specialized state being good $\theta_s = 1$ given its signal realization, i.e.,

$$s = \Pr[\theta_s = 1|s] \in [0, 1]. \tag{4}$$

Note $\int_0^1 s\phi(s)\,ds \equiv q_s$ in order to satisfy prior consistency, where $\phi(s)$ denotes the pdf of s.

General signals being decisive. The specialized Bank A has both general and specialized signals $\{g^A, s\}$ while Bank B only has a general signal g^B. Throughout we assume that the general signal is "decisive" for lending: Bank j bids only if it receives $g^j = H$. Therefore the general signal serves as "pre-screening" for Bank A, i.e., it rejects the borrower upon $g^A = L$ while upon $g^A = H$ it makes a pricing decision based on its specialized signal s. We impose the following parameter restrictions to ensure the pre-screening general signal is decisive.

Assumption 1. ***(Decisive general signals)***

i) Bank A rejects the borrower upon an L general signal, regardless of any specialized signal s:

$$q_g\,(1-\alpha_u)\,\overline{r} < (1-q_g)\,\alpha_d. \tag{5}$$

ii) Bank B is willing to participate (i.e., $r^B < \infty$) if its general signal $g^B = H$:

$$q_g\alpha_u q_s\overline{r} > q_g\alpha_u\,(1-q_s) + (1-q_g)\,(1-\alpha_d)\,; \tag{6}$$

Under Condition (5), the loan is negative NPV to Bank A upon $g^A = L$, even for the most favorable specialized signal $s = 1$. This condition implies that Bank B, which only has the general signal and is uncertain about the realization of the specialized fundamental, also rejects the loan upon receiving $g^B = L$. Condition (6) states that upon $g^B = H$, Bank B is willing to lend at $\overline{r}$ if it is the monopolist lender.

2.3 Discussions on Model Assumptions

There are several model assumptions that are worth discussing further.

Multi-dimensional information structure and its general applications. Our setting with multiple states admits many other interpretations besides general and specialized states. Consider the following multi-dimensional multiplicative setting,

$$\theta = \overbrace{\prod_{n=1}^{\hat{N}} \theta_n}^{\theta_g} \cdot \overbrace{\prod_{n=\hat{N}+1}^{N} \theta_n}^{\theta_s}, \tag{7}$$

with independent binomial states (or characteristics) $\theta_n \in \{0,1\}$ where $n \in \{1,2,...,N\}$; as shown, our model sets $\theta_g \equiv \prod_{n=1}^{\hat{N}} \theta_n$ and $\theta_s \equiv \prod_{n=\hat{N}+1}^{N} \theta_n$. One can always "relabel" to suit the context of a specific application. In a companion paper, He, Huang, and Parlatore (2024) interpret $\prod_{n=1}^{\hat{N}} \theta_n$ and $\prod_{n=\hat{N}+1}^{N} \theta_n$ as the "hard" and "soft" fundamental states, respectively.

Independence between general and specialized states. The assumption that the general state θ_g and the specialized state θ_s are independent is for ease of exposition only. Section 4.3 shows that independence can be relaxed while maintaining tractability. In a companion paper that explores the "span of information" He, Huang, and Parlatore (2024) allows for the two "hard" and "soft" fundamental states to be potentially correlated, which implies the general signals and the specialized signal for Bank A are correlated. For more details, see Section 4.3.

Principal and supplementary signals and comparison to the literature. The equilibrium loan-making rule of the specialized bank is practically relevant. Essentially, the specialized bank has two signals—the general one is "principal" that determines whether to lend, and the other specialized one is "supplementary" which helps its loan pricing.[11] This is in sharp contrast to the existing literature mentioned in the introduction where lenders make loan offers randomly only conditional on the most favorable realization of their (binary) signals. As shown in Section 4.1, our setting—by decoupling the lender's *ex-post* loan assessment from its *ex-ante* technology strength—helps deliver the empirical regularity of lower granted loan rates by specialized banks.

Endogenous information structure. In our main analysis, we take the lenders' information technologies—specifically, Bank A being the specialized lender—as given. Section 4.4 endogenizes this "asymmetric" information technology in a "symmetric" setting with two firms, a and b, where Bank A (B) endogenously becomes specialized by acquiring both "general" and "specialized" signals of the firm a (b), while non-specialized Bank B (A) only acquires the "general" signal of the firm a (b). There, the key difference between these two signals is that a lender j only needs to invest once—say installing IT equipment and software—to get two general signals, one for each firm, while specialized signal needs to be collected individually for each firm.

[11] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit). This ranking portrays the key role played by hard information for large banks when dealing with new borrowers. Indeed, as documented on page 1677 of Crawford, Pavanini, and Schivardi (2018), Italian large banks list the factors they consider in assessing any new loan applicant's creditworthiness, with the following order of importance: i) hard information from the central bank (financial statement data); ii) hard information from Credit Register; iii) statistical-quantitative methods; iv) qualitative information (i.e., bank-specific soft information codifiable as data); v) availability of guarantees; and vi) first-hand information (i.e., branch-specific soft information).

2.4 Credit Market Equilibrium Definition

We now formally define the credit market equilibrium with specialized lending. Before doing so, we define the banks' strategies and their associated profits.

Bank strategies. In equilibrium, each lender makes a potential offer only upon receiving a positive general signal H—recall Assumption 1 guarantees that the general signals are "desicive" for both lenders in making the loan offer or not. Conditional on making offers, we define the space of interest rate offers to be $\mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$. Here, $\overline{r}$ is the exogenous interest rate cap (or, project return) imposed in Section 2.1 and ∞ captures the strategy of not making an offer. The endogenous support of the equilibrium interest rates offered will be a sub-interval of $[0, \overline{r}]$; so with a slight abuse of terminology we refer to that sub-interval as the "support" of the interest rate distribution even though loan rejection ($r = \infty$) could also occur along the equilibrium path.

We denote Bank A's pure strategy by $r^A(s) : [0,1] \to \mathcal{R}$, which induces a distribution of its offers denoted by $F^A(r) \equiv \Pr\left(r^A \leq r\right)$ according to the underlying distribution of the specialized signal. We take as given that Bank A uses pure strategy, though later we formally prove this result in Proposition 1. On the other hand, Bank B randomizes conditional on $g^B = H$, in which case we use $F^B(r) \equiv \Pr\left(r^B \leq r\right)$ to denote the cumulative distribution of its interest rate offers. Because the domain of offers includes rejection $r = \infty$, it is possible that $F^j(\overline{r}) = \mathbb{P}\left(r^j < \infty | g^j = H\right) \leq 1$ for $j \in \{A, B\}$.

The borrower picks the lower interest rate if possible. For instance, conditional on $g^A = g^B = H$, if Bank B quotes r^B, then its winning probability $1 - F^A(r^B)$ equals the probability that Bank A with s offers a rate higher than r^B—note, this includes the event of Bank A with $g^A = H$ but rejecting the borrower ($r^A(s) = \infty$), presumably because of an unfavorable specialized signal. Upon ties $r^A = r^B < \infty$, borrowers randomly choose the lender with probability one half, although the details of the tie-breaking rule do not matter (ties occur as zero-measure events in equilibrium). When $r^A = r^B = \infty$, no bank wins the competition as they both reject the borrower.

Definition 1. (Credit market equilibrium) A competitive equilibrium in the credit market (with decisive general signals) consists of the following lending strategies and borrower choice:

1. A lender j rejects the borrower or $r^j = \infty$ upon $g^j = L$ for $j \in \{A, B\}$; upon $g^j = H$,

   i) Bank A offers $r^A(s) : [0,1] \to \mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$ to maximize its expected lending profits given $g^A = H$ and s, which induces a distribution function $F^A(r) : \mathcal{R} \to [0,1]$;

   ii) Bank B offers $r^B \in \mathcal{R}$ to maximize its expected lending profits given $g^B = H$, which induces a distribution function $F^B(r) : \mathcal{R} \to [0,1]$;

2. Whenever receiving at least one offer, the borrower chooses the lowest offer as long as $\min\{r^A, r^B\} < \infty$.

The following lemma shows that the resulting equilibrium strategies in our setting are still well-behaved as established in the literature (Engelbrecht-Wiggans, Milgrom, and Weber (1983);

Broecker (1990)). The key steps of the proof are standard, though we make certain adjustments due to the presence of both discrete and continuous signals.

Lemma 1. ***(Equilibrium Structure)*** *In any equilibrium, there exists an endogenous lower bound of interest rate $\underline{r} > 0$, so that the two distributions $F^j(\cdot)$, $j \in \{A, B\}$ share a common support $[\underline{r}, \overline{r}]$ (besides ∞ as rejection). Over $[\underline{r}, \overline{r})$ both distributions are smooth, i.e. no gap and atomless, so that they admit well-defined density functions. At most one lender can have a mass point at $\overline{r}$.*

Bank profits and optimal strategies. Denote by $g^A g^B \in \{HH, HL, LH, LL\}$ the event of two general signal realizations, where HL represents Bank A's (B's) general signal being H (L). Denote by $p_{g^A g^B}$ the joint probability of any collection of realizations of general signals; e.g., $p_{HH} \equiv \mathbb{P}\left(g^A = H, g^B = H\right) = q_g \alpha_u^2 + (1 - q_g)(1 - \alpha_d)^2$. Similarly, denote by $\mu_{g^A g^B} \equiv \mathbb{P}\left(\theta_g = 1 \middle| g^A, g^B\right)$ the posterior probability of the general state being one conditional on $g^A g^B$; for instance,

$$\mu_{HH} = \frac{q_g \alpha_u^2}{q_g \alpha_u^2 + (1 - q_g)(1 - \alpha_d)^2}.$$

And, since $\{\theta_g, \theta_s\}$ are independent, the posterior of project success given $\{HH, s\}$ is

$$\mathbb{P}\left(\theta = 1 | \, g^A = H, g^B = H, s\right) = \mu_{HH} \cdot s. \tag{8}$$

For Bank A who receives $g^A = H$ and s, its profit $\pi^A(r|s)$ by quoting $r \in [\underline{r}, \overline{r}]$ equals

$$\pi^A(r|s) \equiv \underbrace{p_{HH}}_{g^A = g^B = H} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} [\mu_{HH} s (1 + r) - 1] + \underbrace{p_{HL}}_{g^A = H, g^B = L} [\mu_{HL} s (1 + r) - 1]. \tag{9}$$

Bank A can also choose to exit by quoting $r = \infty$, in which case $\pi^A(\infty|s) = 0$. We then denote Bank A's optimal interest rate offer by

$$r^A(s) \equiv \arg\max_{r \in \mathcal{R}} \pi^A(r|s).$$

To understand Eq. (9), recall that Bank A cannot observe g^B when making an offer. With probability p_{HH}, both banks get favorable general signals and Bank A quoting r wins with probability $1 - F^B(r)$, whereas with probability p_{HL} it faces no competition as Bank B with $g^B = L$ withdraws itself. Standard winner's curse logic implies that whether Bank B participates in the loan market affects Bank A's perceived borrower quality (regarding the general fundamental state) captured by μ_{HH} or μ_{HL}. Importantly, since Bank B randomizes its strategy upon $g^B = H$, from the perspective of Bank A winning the price competition against Bank B is not informative about borrower quality.

This last observation is in sharp contrast with the problem of the non-specialized Bank B, who understands that the outcome of competition against its specialized opponent is informative about θ_s. More specifically, besides the possibility of the competitor's unfavorable general information as mentioned above, the non-specialized lender B knows that $r^A(s) > r^B$ upon winning the competition. Because the equilibrium $r^A(s)$ is decreasing, an important equilibrium property that we will verify later, upon winning Bank B infers an unfavorable specialized signal. Taking these inferences into account, Bank B's lending profit when quoting r is

$$\pi^B(r) \equiv \underbrace{p_{HH}}_{g^A=g^B=H} \underbrace{\left[1-F^A(r)\right]}_{B \text{ wins}} \mathbb{E}\left[\mu_{HH}\theta_s(1+r)-1 \middle| r \le r^A(s)\right] + \underbrace{p_{LH}}_{g^A=L,g^B=H} \left[\mu_{LH}q_s(1+r)-1\right]. \quad (10)$$

Bank B's optimal strategy $F^B(\cdot)$ maximizes its expected payoff

$$\max_{F^B(\cdot)} \int_{\mathcal{R}} \pi^B(r)\, dF^B(r). \quad (11)$$

As it is standard in equilibria in mixed strategies, the profit-maximizing Bank B is indifferent between any r on its support.

3 Credit Market Equilibrium Characterization

To characterize the credit market equilibrium, in Section 3.1 we first take the equilibrium non-specialized Bank B's profit π^B as given and solve for the other equilibrium objects. We then solve for π^B in Section 3.2, and Section 3.3 completes the construction of the credit market equilibrium.

3.1 Solving for the Pricing Strategies of the Lenders

Solving for $r^A(s)$. Following Milgrom and Weber (1982), we start by solving for Bank A's equilibrium strategy $r^A(s)$. Suppose that $r^A(s)$ is decreasing, which we shall verify later. Bank B who plays mixed strategies must make a constant profit $\pi^B \ge 0$ from any interest rate along the equilibrium support. When Bank B rejects the borrower upon H with some probability we must have $\pi^B = 0$. Our goal is to characterize both lenders' strategies by taking π^B as given.

Conditional on $g^A = H$, when Bank B quotes $r = r^A(s)$, it wins the borrower only when A's specialized signal is below s. Bank B, therefore, updates its beliefs about the borrower's quality accordingly—its posterior for the specialized state is $\int_0^s t\phi(t)\,dt$. On the other hand, conditional on $g^A = L$, Bank B wins the borrower for sure. Plugging $r^B = r^A(s)$ in Bank B's lending profits in Eq. (10), we have the following indifference condition:

$$\pi^B = \underbrace{\left[p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right]}_{B\text{'s expected loan quality (lending benefit)}} \left(1+r^A(s)\right) - \underbrace{\left(p_{HH}\Phi(s) + p_{LH}\right)}_{B\text{'s expected loan size (lending cost)}}. \quad (12)$$

Eq. (12) holds for any $r^A(s) \in [\underline{r}, \overline{r})$, which implies that

$$r^A(s) = \frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s} - 1. \quad \text{when} \quad s \in [\hat{s}, 1]. \quad (13)$$

where $\hat{s}$ is the highest specialized signal realization so that Bank A quotes $\bar{r}$:

$$\hat{s} \equiv \sup \left\{ s | \, r^A(s) = \bar{r} \right\}. \tag{14}$$

We discuss several relevant boundaries in Bank A's equilibrium strategy based on Eq. (13). First, because Bank A with the highest specialized signal $s = 1$ quotes $\underline{r}$ in equilibrium, we can derive $\underline{r}$ as an affine function of π^B by setting $s = 1$ in Eq. (13):

$$\underline{r} = r^A(1) = \frac{\pi^B + p_{HH} + p_{LH}}{(p_{HH}\mu_{HH} + p_{LH}\mu_{LH})q_s} - 1 \Leftrightarrow \pi^B = \frac{(1+\underline{r}) \cdot (p_{HH}\mu_{HH} + p_{LH}\mu_{LH})q_s}{p_{HH} + p_{LH}}. \tag{15}$$

Intuitively, in equilibrium, both lenders share the same endogenous lower bound $\underline{r}$. From the perspective of Bank B, quoting $\underline{r}$ guarantees winning and $h^B = H$ is the only information without any inference from competition. Therefore Bank B's profit is determined by the expected loan quality conditional on $h^B = H$, i.e., its gross rate offer $1 + \underline{r}$ multiplied by the probability of good borrower $(p_{HH}\mu_{HH} + p_{LH}\mu_{LH})q_s$ and scaled by the lending probability $p_{HH} + p_{LH}$.

Second, we define $x \leq \hat{s}$ as the threshold such that $\pi^A(\bar{r}| \, x) = 0$. It is straightforward to show that $r^A(s) = \bar{r}$ for $s \in [x, \hat{s})$, and $r^A(s) = \infty$ for $s \in [0, x)$. Note that $x = \hat{s}$ can occur in equilibrium (which, as we will show, occurs when $\pi^B > 0$).

Proposition 1 below shows that Bank A's strategy $r^A(s)$ is always decreasing in equilibrium (no ironing needed). Define its inverse function (correspondence) of $r^A(s)$ to be

$$s^A(r) \equiv \begin{cases} r^{A(-1)}(r), & \text{when } r \in [\underline{r}, \bar{r}), \\ [x, \hat{s}), & \text{when } r = \bar{r}, \\ [0, x), & \text{when } r = \infty. \end{cases} \tag{16}$$

Here, we take the convention that $r^A(x) = \bar{r}$. The two relevant cutoffs for Bank A's strategy can be written as $\hat{s} = \sup s^A(\bar{r})$, i.e., the highest signal that Bank A quotes $\bar{r}$, and $x = \sup s^A(\infty)$, i.e, the highest signal under which Bank A rejects the borrower.

Solving for $F^B(\cdot)$. Recall Bank B is indifferent among all rates on the support. In equilibrium, B's strategy is pinned down to support $r^A(\cdot)$ in Eq. (13) as A's optimal strategy. The first-order-condition (FOC) that maximizes Bank A's objective in Eq. (9), which balances the lower probability of winning against the higher payoff from served borrowers, is

$$p_{HH}\left(-\frac{dF^B(r)}{dr}\right)[\mu_{HH}s(1+r) - 1] + \left\{p_{HH}\left[1 - F^B(r)\right]\mu_{HH}s + p_{HL}\mu_{HL}s\right\} = 0. \tag{17}$$

That Bank A's equilibrium strategy $r^A(\cdot)$ in Eq. (13) satisfies Eq. (17) helps us pin down $F^B(\cdot)$, which, as we now show, solves a simple differential equation. To this end, we first consider Bank B who maximizes the expression in Eq. (12) and understands the corresponding marginal borrower type (with a specialized signal) is $s^A(r)$ when quoting r. Writing everything in terms of

r; when Bank B marginally cuts its quote by dr, it gets $\phi\left(s^A(r)\right)(-s^{A\prime}(r))dr$ additional borrower type with quality $\mu_{HH}s^A(r)$ if there is competition, which occurs with probability p_{HH}. This gain should equal to the marginal lower payoff from the borrower types who are already served, implying Bank B's FOC (12) to be

$$\underbrace{p_{HH}\left[\phi\left(s^A(r)\right)\cdot(-s^{A\prime}(r))\right]}_{\text{additional borrower type}}\left[\mu_{HH}s^A(r)(1+r)-1\right]=\underbrace{p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi(t)\,dt+p_{LH}\mu_{LH}q_s}_{\text{existing borrower types}}. \tag{18}$$

Solving for $p_{HH}\left[\mu_{HH}s^A(r)(1+r)-1\right]$ using Bank B's FOC in Eq. (18) and plugging it in Eq. (17) which captures Bank A's FOC, we have

$$\frac{dF^B(r)}{dr}\left[\frac{p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi(t)\,dt+p_{LH}\mu_{LH}q_s}{\phi\left(s^A(r)\right)s^{A\prime}(r)}\right]+p_{HH}\left[1-F^B(r)\right]\mu_{HH}s^A(r)+p_{HL}\mu_{HL}s^A(r)=0.$$

One can show that the above equation yields the following ordinary differential equation (ODE), which pins down $F^B(\cdot)$:

$$\frac{d}{dr}\left\{\frac{p_{HH}\mu_{HH}\left[1-F^B(r)\right]+p_{HL}\mu_{HL}}{p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi(t)\,dt+p_{LH}\mu_{LH}q_s}\right\}=0. \tag{19}$$

The intuition behind the ODE in Eq. (19) is as follows. At any interest rate r, both lenders are competing for the same marginal borrower type with an expected profit of $\mu_{HH}\cdot s^A(r)\cdot(1+r)-1$.[12] Denote by $Q^j(r)$ the total *q*uality of borrowers of Bank j when offering interest rate r. Then,

$$\begin{aligned}Q^A(r)&=p_{HH}\mu_{HH}s^A(r)\left[1-F^B(r)\right]+p_{HL}\mu_{HL}s^A(r),\\Q^B(r)&=p_{HH}\mu_{HH}\int_0^{s^A(r)}t\phi(t)\,dt+p_{LH}\mu_{LH}q_s.\end{aligned}$$

Q^A and Q^B differ in that A observes s while B only knows that it gets borrower types with $s<s^A(r)$ (if $g^A=H$) or q_s (if $g^A=L$). For Bank A, the marginal effect of price cutting on borrower quality is $\frac{1}{\mu_{HH}}\left[\frac{Q^A(r)}{s^A(r)}\right]'$, where the division inside the bracket adjusts for the quality of the specialized fundamental of the marginal borrower type. Then, Bank A's optimal pricing strategy equates the above marginal benefit to the associated marginal cost of price cutting, which is dr

[12] This term shows up in both optimization conditions, i.e., (17) for Bank A and (18) for Bank B).

multiplied by the expected borrower quality $Q^A(r)$. Therefore we must have

$$\underbrace{\left[\frac{Q^A(r)}{\mu_{HH}s^A(r)}\right]' dr \cdot \left[\mu_{HH}s^A(r)(1+r)-1\right]}_{\text{MB on marginal borrower type}} = \underbrace{Q^A(r)dr}_{\text{MC on existing borrower types}}$$

$$\Leftrightarrow \frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)(1+r)-1} = \frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}}, \tag{20}$$

which is equivalent to (17). On the other hand, for Bank B the marginal effect on borrower quality is $\frac{1}{\mu_{HH}}\frac{Q^{B\prime}(r)}{s^A(r)}$,[13] implying an optimality condition of

$$\underbrace{\frac{Q^{B\prime}(r)}{\mu_{HH}s^A(r)}dr \cdot \left[\mu_{HH}s^A(r)(1+r)-1\right]}_{\text{MB on marginal borrower type}} = \underbrace{Q^B(r)dr}_{\text{MC on existing borrower types}} \Leftrightarrow \frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)(1+r)-1} = \frac{Q'^B(r)}{Q^B(r)}. \tag{21}$$

This is exactly (18). Combining (20) and (21), we derive the key ODE in Eq. (19):

$$\frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}} = \frac{Q'^B(r)}{Q^B(r)} \Leftrightarrow \frac{d}{dr}\left[\frac{Q^A(r)/s^A(r)}{Q^B(r)}\right] = 0. \tag{22}$$

Two additional pieces help us solve for $F^B(\cdot)$ based on (19). First, $F^B(\underline{r}) = 0$, which says that Bank B never offers rates below the endogenous lower-end support, gives the boundary condition. Second, we have derived $\underline{r}(\pi^B)$ in (15) as a function of π^B. Focusing on the interior of the strategy space, we have:

$$1 - F^B(r) = \frac{\int_0^{s^A(r)} t\phi(t)\,dt}{q_s}, \text{ for } r \in (\underline{r}, \overline{r}) \tag{23}$$

It is clear that $F^B(r) < 1$ for $r \in [\underline{r}, \overline{r})$, because $F^B(\overline{r}^-) = \frac{1}{q_s}\int_{s^A(\overline{r}^-)=\hat{s}}^1 t\phi(t)dt < 1$; and Bank B's strategy on the boundary $\overline{r}$ depends on whether it is profitable in equilibrium. More precisely, it either places a mass of $1 - F^B(\overline{r}^-) = \frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt > 0$ on $\overline{r}$ if $\pi^B > 0$, or quotes $r = \infty$ (i.e., withdraws) if $\pi^B = 0$.[14]

Illustration of lenders' pricing strategies. Figure 2 illustrates the equilibrium strategies for both lenders for two cases, $\pi^B > 0$ and $\pi^B = 0$ indicated by the subscripts "+" and "0," respectively. The exogenous parameter that drives the different profits for Bank B is $\overline{r}$, which we

[13] Readers might notice the important difference between the two lenders' marginal effects of cutting their prices on the quantity. For Bank A who observes the specialized signal realization directly, its pricing decision should not affect its quality; this is why we scale Q^A first by s and then take derivative, i.e., $\left[\frac{Q^A(r)}{s^A(r)}\right]'$. In contrast, without observing s directly, Bank B's price cutting affects its inferred quality of the borrower type (that it wins over Bank A). Therefore we take the derivative of $Q^B(r)$, which includes the quality of its borrowers, and then scale by the quality of marginal borrower type to avoid double counting.

[14] Although the information technology parameters on the general signals do not enter $F^B(\cdot)$ in (23) directly, they affect $F^B(\cdot)$ indirectly via the endogenous lower bound of the support $\underline{r}$.

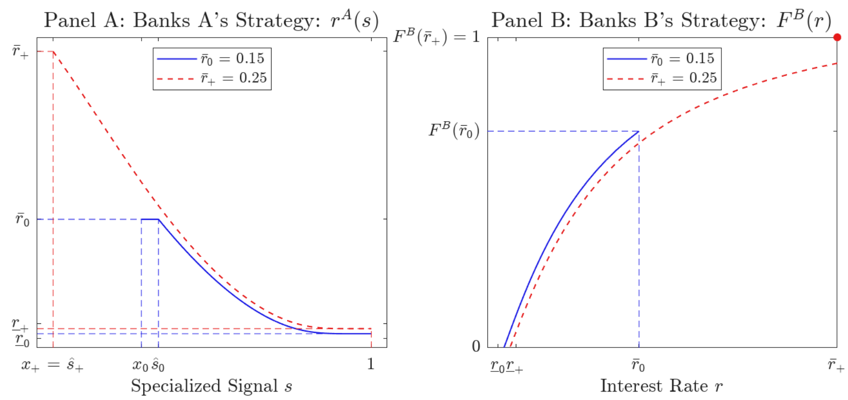

Figure 2: **Equilibrium strategies $r^A(s)$ for Bank A (left) and $F^B(r)$ for Bank B (right).** In both panels, strategies under $\bar{r}_+$ (i.e., positive-weak equilibrium) are depicted in red dashed lines while strategies with $\bar{r}_0$ (i.e., zero-weak equilibrium) are depicted in blue solid lines. In the zero-weak equilibrium, Bank A (but not Bank B) has a point mass at $\bar{r}_0$ while in the positive-weak equilibrium, Bank B (but not Bank A) has a point mass at $\bar{r}_+$. Parameters: $q_g = 0.8$, $q_s = 0.95$, $\alpha_u = \alpha_d = \alpha = 0.9$, and $\tau = 0.5$, where τ captures the signal-to-noise ratio of Bank A's specialized information technology as $s = \mathbb{E}[\theta_s|\theta_s + \epsilon]$ and $\epsilon \sim \mathcal{N}(0, 1/\tau)$.

denote respectively by $\bar{r}_+$ and $\bar{r}_0$, where $\bar{r}_+ > \bar{r}_0$. As one would expect, the, where greater the borrower surplus $\bar{r}$, the higher the lender's profits. Panel A (left) depicts Bank A's pricing strategy $r^A(s)$, which is decreasing, while the right panel plots Bank B's CDF of its rates $F^B(r)$. We also plot the two signal cutoffs—$\hat{s}$, at which Bank A's strategy hits $\bar{r}$, and x, at which Bank A exits.

Figure 2 highlights a key difference between the two types of equilibrium that can arise, one with $\pi^B = 0$—we call it the zero-weak equilibrium as the weak bank earns no profits—and the other with $\pi^B > 0$—we call it the positive-weak equilibrium as the weak bank makes positive profits. As shown, if $\pi^B = 0$ Bank A has a point mass at $\bar{r}_0$ (corresponding to $s \in (x_0, \hat{s}_0)$) but Bank B does not, while if $\pi^B = 0$ the opposite holds. This reflects the fierce competition at the interest rate cap and it is the exact manifestation of the last point in Lemma 1 (otherwise, lenders will undercut each other at this point).

3.2 Solving for the Equilibrium Profit of Bank B

We now solve for Bank B's equilibrium profit which pins down the entire equilibrium given the results in Section 3.1. First, define s_A^{be} to be the specialized signal under which Bank A quotes $\bar{r}$ and breaks even (therefore the superscript "be"). Formally, using $\pi^A(\cdot)$ given in (9) and using the strategic response of Bank B in Eq. (23), s_A^{be} is the unique solution to the following equation

$$0 = \pi^A\left(\bar{r}\,\middle|\,s_A^{be}\right) = p_{HH}\frac{\int_0^{s_A^{be}} t\phi(t)\,dt}{q_s}\cdot\left[\mu_{HH}s_A^{be}(1+\bar{r})-1\right] + p_{HL}\left[\mu_{HL}s_A^{be}(1+\bar{r})-1\right]. \quad (24)$$

As shown in footnote 15, (24) admits a unique solution inside the interval $(0,1)$. We can define s_B^{be} similarly; taking B's payoff function in (12) and setting r to $\overline{r}$, s to s_B^{be}, and π^B to zero give us:[15]

$$0 = \pi^B\left(r = \overline{r}; s = s_B^{be}\right) = p_{HH}\left[\mu_{HH}\left(\int_0^{s_B^{be}} t\phi(t)dt\right)(1+\overline{r}) - \Phi\left(s_B^{be}\right)\right] + p_{LH}\left[\mu_{LH}q_s(1+\overline{r}) - 1\right]. \tag{25}$$

Lemma 2 below shows that the relative ranking between s_B^{be} and s_A^{be} fully determines π^B and $\hat{s}$. Intuitively, the equilibrium type crucially depends on which lender—when quoting $\overline{r}$—hits zero profit first as s decreases. If $s_A^{be} < s_B^{be}$ then Bank B hits zero profit first, and this supports the equilibrium with $\pi^B = 0$ with $\hat{s} = s_B^{be}$; otherwise we have $\pi^B > 0$ with $\hat{s} = s_A^{be}$.

Lemma 2. *Given s_A^{be} defined in (24), the equilibrium Bank B profit is*

$$\pi^B = \max\left\{\left[p_{HH}\mu_{HH}\int_0^{s_A^{be}} t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right](1+\overline{r}) - \left(p_{HH}\Phi\left(s_A^{be}\right) + p_{LH}\right), 0\right\}.$$

When $s_B^{be} < s_A^{be}$ we are in the positive-weak equilibrium in which the weak Bank B makes a positive profit, and $x = \hat{s} = s_A^{be}$. Otherwise, when $s_B^{be} \geq s_A^{be}$ we are in the zero-weak equilibrium where Bank B earns zero profits, with $x < \hat{s} = s_B^{be}$.

3.3 Credit Market Equilibrium

Credit market equilibrium characterization. The next proposition provides the full analytical characterization of the credit market equilibrium with specialized lending.

Proposition 1. ***(Credit Market Equilibrium)*** *In the unique equilibrium, Bank A follows a pure strategy as in Definition 1. In this equilibrium, lenders reject the borrower upon a low general signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows, with the equilibrium π^B given in Lemma 2.*

1. *Bank A with a specialized signal s offers*

$$r^A(s) = \begin{cases} \min\left\{\frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH}\int_0^s t\phi(t)dt + p_{LH}\mu_{LH}q_s} - 1, \overline{r}\right\} & \text{for} \quad s \in [x, 1], \\ \infty, & \text{for} \quad s \in [0, x). \end{cases} \tag{26}$$

   The equation pins down $\underline{r} = r^A(1)$. If $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\overline{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

[15] There are several points to make. Regarding the definition of s_B^{be}, the rate r in (12) is $r^A(s)$; we essentially separate rate r and s in (25). Regarding s_A^{be}, technically speaking in (24) Bank A quotes $\overline{r}^-$ so that $1 - F^B\left(\overline{r}^-\right) = \frac{1}{q_s}\int_0^{s_A^{be}} t\phi(t)\,dt$, as (23) requires $r \in [\underline{r}, \overline{r})$. Second, we have a unique solution of (24) because $\pi^A\left(\overline{r}\,\middle|\,s_A^{be}\right)$ is strictly increasing in s_A^{be}, with $\pi^A\left(\overline{r}\,\middle|\,s_A^{be} = 0\right) < 0$ and $\pi^A\left(\overline{r}\,\middle|\,s_A^{be} = 1\right) = p_{HH}\left[\mu_{HH}(1+\overline{r}) - 1\right] + p_{HL}\left[\mu_{HL}(1+\overline{r}) - 1\right] > 0$; the latter is implied by Bank A's willingness to make an offer given $g^A = H$.

2. *Bank B makes an offer with cumulative probability given by*

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s} & \text{for} \quad r \in [\underline{r}, \overline{r}) \\ 1 - \mathbf{1}_{\{\pi^B=0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} & \text{for} \quad r = \overline{r} \end{cases}, \tag{27}$$

where $\mathbf{1}_{\{X\}} = 1$ *if* X *holds and is zero otherwise. When* $\pi^B = 0$*,* $F^B(\overline{r}) = F^B(\overline{r}^-)$ *is the probability that Bank B makes the offer (and with probability* $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ *it withdraws by quoting* $r^B = \infty$*); when* $\pi^B > 0$*,* $F^B(\overline{r}) = 1$ *and there is a point mass of* $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\,dt$ *at* $\overline{r}$*.*

The proof for Proposition 1 mainly covers three theoretical issues. First, we show that the specialized lender always adopts a pure strategy in any equilibrium; that is to say, Bank A's pure strategy, which is implicitly taken as given in Definition 1, is a result rather than an assumption. Second, we prove that the FOC conditions used in the equilibrium construction detailed in Section 3 are sufficient to ensure global optimality. Third, somewhat surprisingly, thanks to the endogenous adjustment of π^B and $\underline{r}$, we never need to "iron" a la Myerson (1981) in the interior range for equilibrium interest rates. In fact, consistent with point 3 in Lemma 1, Bank A never bunches its quotes—except at the exogenous rate cap $\overline{r}$ when the zero-weak equilibrium ensues.

Properties of credit market equilibrium. Figure 3 illustrates the main properties of the credit market equilibrium with specialized lenders. For exposition purposes, we assume that Bank A's specialized signal s is obtained from observing $\theta_s + \epsilon$, so that

$$s = \mathbb{E}[\theta_s | \theta_s + \epsilon], \tag{28}$$

where $\epsilon \sim \mathcal{N}(0, 1/\tau)$ indicates a white noise, with precision parameter τ, which captures the signal-to-noise ratio of Bank A's specialized information technology.

The top two panels in Figure 3 plot both lenders' pricing strategies conditional on making an offer; Panel A is the same as that in Figure 2 for convenience while Panel B plots the density dF^B/dr for Bank B.

Formally, we call Bank A's strategy of $r^A(s)$ decreasing in s "private-information-based pricing," which has important implications on the equilibrium interest rate differentials studied in Section 4.1. When Bank A's private assessment of borrower quality is sufficiently low, i.e., $s < x$, it rejects the borrower. Panel C further plots the two specialized signal cut-offs for Bank A, i.e., $\hat{s}$ at which it starts quoting $\overline{r}$ and x at which it starts rejecting the borrower.

Finally, Panel D plots the expected profits—$\mathbb{E}(\pi^A)$ and π^B—for the two lenders, against the exogenous interest rate cap $\overline{r}$. Recall that $\overline{r}$ can also be interpreted as the return of a good project, capturing the surplus to be realized from a loan. Thus, a higher total surplus gives rise to less fierce competition, and as a result, both lenders—including the weak lender B—make positive expected profits upon a favorable general signal H. This explains Panel D, which shows that π^B is strictly positive for sufficiently high values of $\overline{r}$. Put differently, the model features a positive-(zero-) weak equilibrium when $\overline{r}$ is relatively high (low).

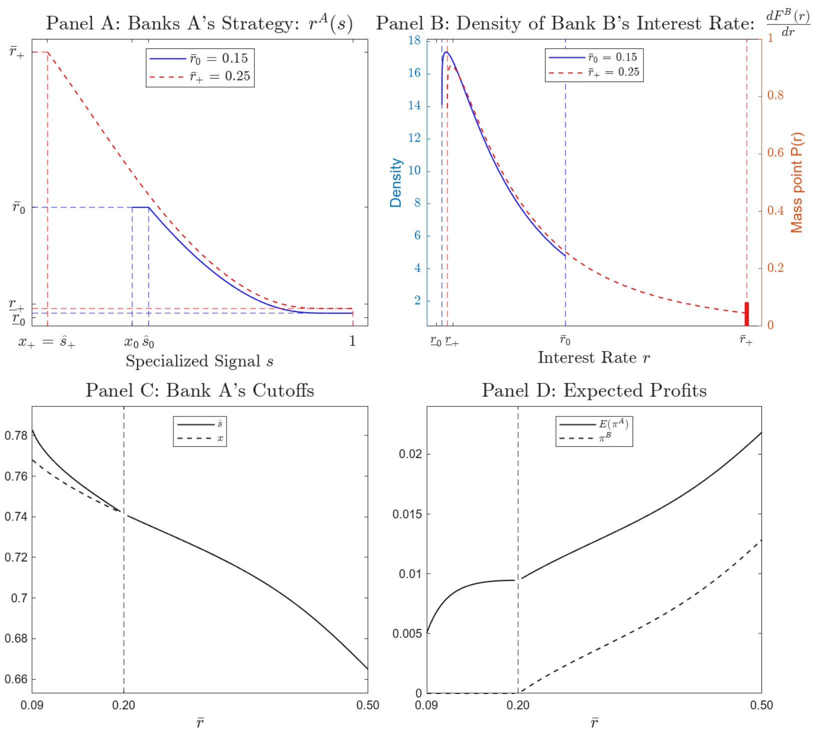

Figure 3: **Equilibrium strategies and profit.** In the top two panels, we plot equilibrium strategies for both lenders. Panel A depicts $r^A(s)$ as a function of s and Panel B plots $dF^B(r)/dr$ for as a function r; strategies with $\bar{r}_+$ are depicted in red dashed lines while strategies with $\bar{r}_0$ are depicted in blue solid lines. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders. Parameters: $q_g = 0.8$, $q_s = 0.95$, $\alpha_u = \alpha_d = \alpha = 0.9$, and $\tau = 0.5$, where τ captures the signal-to-noise ratio of Bank A's specialized information technology as $s = \mathbb{E}[\theta_s|\theta_s + \epsilon]$ and $\epsilon \sim \mathcal{N}(0, 1/\tau)$.

For a better illustration, consider the competition at interest rate $\bar{r}$. In the positive-weak equilibrium (high $\bar{r}$'s), the non-specialized Bank B has a point mass on this interest rate, enjoying some "local monopoly power" in competition as it is the only lender in the market when Bank A rejects the borrower upon $s < \hat{s} = x$. This is possible because when the project's surplus (captured by $\bar{r}$) is sufficiently large, the non-specialized lender B is still profitable by quoting $\bar{r}$ despite the winner's curse. We highlight that the weak lender's profits come from its conditionally independent private signal, which could also arise in canonical models; the weak lender's "local monopoly power," however, is a unique feature of our model that arises from Bank A's informed pricing to withdraw. (This point will be elaborated on in footnote 19 when we discuss the "private-information-bsaed pricing effect.") In contrast, in the zero-weak equilibrium (low $\bar{r}$'s), the specialized Bank A, with a point mass at $\bar{r}$ (when $s \in (x, \hat{s})$, as shown in Panel C), is the monopolistic lender while the nonspecialized Bank B withdraws.

4 Model Implications and Extensions

We now discuss the economic implications of our model. First, we study the model implied interest rate wedge, defined as the difference between the rates of loans made by specialized and non-specialized lenders; we highlight the difference between bids and winning bids on granted loans. We then explain how our private-information-based pricing helps generate a negative interest rate wedge, an empirical fact for which we offer detailed evidence based on Y-14 supervisory data. We then show our theoretical results are robust to a generalized information structure, and finally endogenize the bank specialization structure that we have assumed so far.

4.1 Specialized Lending: Interest Rate Wedge

As suggested by Figure 1 (and thoroughly established in Section 4.2), the loans on the balance sheets of specialized lenders tend to have higher quality and lower interest rates. Specialized lenders with informational advantage are extending higher quality loans in our model, which is a robust prediction of any information-based model including those canonical ones a la Broecker (1990) and Marquez (2002). In what follows, we thus focus on the model implications on interest rates.

Interest rate wedge: bids vs. winning bids

An econometrician observes the granted bank loans accepted by borrowers. Put differently, the loans we use to calculate loan interest rates are already on the book of the lender who has won the bidding competition. In our setting, when Bank A makes a loan offer ($r^A < \infty$), it is accepted by the borrower if $r^A < r^B \leq \infty$—either if there is no offer from Bank B (e.g., when $h^B = L$ so $r^B = \infty$) or Bank A's rate is lower than that offered by Bank B. Therefore, the theoretical counterpart of negative rate differentials in Figure 1 is

$$\Delta r \equiv \underbrace{\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right]}_{\text{interest rate of } A\text{'s granted loan}} - \underbrace{\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]}_{\text{interest rate of } B\text{'s granted loan}} < 0, \tag{29}$$

where $\{r^i < r^j \leq \infty\}$ denotes the event that Bank i wins the competition.

We call Δr in (29) the interest rate wedge. There is a crucial difference between the interest rate wedge calculated from "bids," i.e., banks' offered interest rates, and the one calculated from "winning bids," i.e., banks' rates on their granted loans. The winning bid is a first-order statistic (i.e., the smaller one given two quotes); and in our context of lending competition banks may simply reject loan applications by quoting ∞ due to winner's curse. Therefore the winning bid necessarily requires $r^i < \infty$, which is implied by the conditioning in Eq. (29).

In a nutshell, although the winner's curse pushes the less informed Bank B to bid higher (often in the form of withdrawals by quoting $r = \infty$), the exact force leads to higher winning bids from the more informed Bank A. An example from He, Huang, and Zhou (2023) illustrates this point

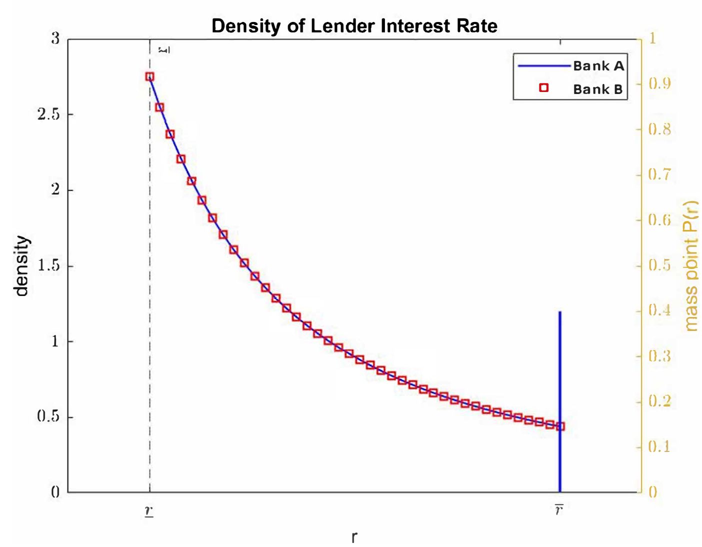

Figure 4: **Example of Lender Bidding Rates in Canonical Models.** We plot the density functions (left scale) and probability mass points (right scale) of lenders' interest rate offer upon favorable signals in He, Huang, and Zhou (2023). Our framework nests He, Huang, and Zhou (2023) by setting $q_s = 1$ (degenerate specialized information), $\alpha_A^u = \alpha_B^u = 1$ (bad-news information structure) and $\alpha_A^d > \alpha_B^d$ (Bank A has better information technology). Here, the endogenous lower bound for rates is $\underline{r} \equiv (1-q)\left(1-\alpha_d^B\right)/q$.

starkly. There, banks are endowed with general signals only, and for simplicity they assume a bad news structure, i.e., $\alpha_u^j = 1$ and $\alpha_d^j < 1$ so that only false positives can occur. Banks differ in the precision of their signals; to capture the idea of specialization, suppose that $\alpha_d^A > \alpha_d^B$ so Bank A is relatively more informed. And for illustration purpose, our analysis is conditional on both lenders making offers (i.e., upon two H general signals).

As shown in He, Huang, and Zhou (2023), the equilibrium CDF of offered rates for both banks, denoted by $\hat{F}(\cdot)$, coincides in the interior of the common support $r \in [\underline{r}, \overline{r})$, with

$$\hat{F}(r) \equiv \mathbb{P}(\tilde{r}_A < r) = \mathbb{P}(\tilde{r}_B < r) = \frac{r - \frac{1-q}{q}\left(1-\alpha_d^B\right)}{r - \frac{1-q}{q}\left(1-\alpha_d^B\right)\left(1-\alpha_d^A\right)}.$$

Figure 4 plots the bidding strategies of both lenders in He, Huang, and Zhou (2023); as shown, their densities coincide in the interior of the support. The only difference in the lenders' strategies is at the upper boundary $\overline{r}$: Bank A quotes the monopolistic rate $r = \overline{r}$ with a positive mass $1 - \hat{F}(\overline{r}^-) > 0$ while Bank B rejects the borrower by quoting $r = \infty$ with the same probability. Consistent with the intuition of the winner's curse, the bidding rates from the less informed Bank B are higher than those from the more informed Bank A (i.e., first-order stochastic dominance, because $\infty > \overline{r}$). However, one can formally show that the interest rate wedge calculated from winning bids goes the opposite way—in these events, Bank A earns a monopolistic profit of $r^A = \overline{r}$ (which is counted in the winning bids) while Bank B rejects (quoting $r^B = \infty$ which is not counted in the winning bids).[16]

[16] Recall that this discussion only concerns the event of participation from both lenders. In this case, one can

Because Bank A's monopoly rent comes from its information advantage, we call this economic force the "information rent" effect. Combining this with the "private information-based pricing" effect discussed in Section 3.3 (right after Figure 3), the next section offers some formal theoretical results on the interest rate wedge.

Economic mechanisms determining the interest rate wedge

Canonical models: the information rent effect. Canonical credit market competition models parameterize the information technology by the signal's precision that captures the lenders' ability to screen out uncreditworthy borrowers. There, the most natural way to capture "specialized lending" is by imposing asymmetric screening abilities on general signals (assuming a degenerate specialized fundamental state which always equals one) along the line of Marquez (2002); He, Huang, and Zhou (2023), as illustrated in Figure 4.

Regarding the specific information structure of general signals given in (3)), the literature has primarily focused on the following two parameterizations. The first is the bad news structure adopted in He, Huang, and Zhou (2023) assuming that $\alpha_d^A > \alpha_d^B$ (and $\alpha_u^A = \alpha_u^B = 1$), based on which we produce Figure 4. Alternatively, Marquez (2002) and He, Jiang, and Xu (2024) adopt a symmetric information structure in which $\alpha_u^A = \alpha_d^A > \alpha_u^B = \alpha_d^B$. In the bad news structure, Bank A makes fewer false positive mistakes than Bank B, while in the symmetric information structure, A makes fewer false positive and false negative mistakes than B. For ease of exposition, in both cases, we use $\alpha^A > \alpha^B$ to denote Bank A having a more informative (binary) signal.

As emphasized before, in these canonical models only quantity decisions (i.e., whether to lend or not) are based on the signal realizations while pricing decisions (offered interest rates) are randomized. We have the following proposition.

Proposition 2. ***(Counterfactual Prediction in Canonical Models.)***

1. *Under a bad news structure, there exists a threshold $\hat{\bar{r}}$ such that $\Delta r > 0$ for $\bar{r} < \hat{\bar{r}}$;*
2. *Under a symmetric information structure, when $\alpha = \alpha^A$ and $\alpha^B \uparrow \alpha$, $\Delta r > 0$ for $\bar{r} \leq \frac{1}{q} - 1$ or $q \geq 1 - \alpha + \alpha^2$.*

In general, as Bank A's private signal is more precise, the weak lender B is more concerned about the winner's curse, i.e., picking up a "lemon" whom the competitor lender rejected. As a result, B randomly withdraws even after receiving a favorable signal $g^B = H$, effectively making Bank A a monopolist. This exactly corresponds to the *information rent* effect, mentioned right after Figure 3, driving the specialized Bank A to have higher expected winning bids (i.e., rates on granted loans) than Bank B. This force pushes the model to deliver a negative interest wedge.

formally prove that $\Delta r > 0$. However, from an unconditional perspective, we also need to take into account the possibility of an unfavorable general signal under which each lender quotes $r = \infty$. Given a bad news structure, the stronger Bank A is more likely to receive an unfavorable general signal (which is truth-revealing) and therefore reject the loan. This force complicates the analysis and we show in Proposition 2 that $\Delta r > 0$ when $\bar{r}$ is sufficiently small (i.e., when loan rejection occurs often in equilibrium). Nevertheless, when discussing the result in Proposition 2 we point out that the threshold of $\bar{r}$ is too large to be relevant in practice.

The first part of Proposition 2 concerns the bad news structure; there, the information rent effect intensifies if the weak lender rejects borrowers more often in the equilibrium. The lower the exogenous interest rate cap $\bar{r}$, the more severe the winner's curse, and therefore the weak lender is more likely to reject its loan applications. This explains the first part of Proposition 2, which says that the canonical models generate $\Delta r > 0$ as long as $\bar{r}$ is sufficiently low. The threshold $\hat{\bar{r}}$ for the rate cap, under empirically relevant parameters calibrated based on U.S. banking data, is way above 36%, the rate cap imposed by most U.S. usury laws.[17] We, therefore, conclude that the model predictions under the bad news structure are counterfactual in light of Figure 1.

The second part of Proposition 2 considers the symmetric information case. We are unable to provide a proof for the general case; instead, we consider the limiting case of $\alpha^B \uparrow \alpha^A$ and show that under empirically relevant primitives calibrated in Appendix A.3,[18] we would have the counterfactual prediction $\Delta r > 0$ even when $\alpha^B \uparrow \alpha^A$. Presumably, the information rent effect is stronger when the gap in information technology $\alpha^A - \alpha^B > 0$ is larger, which is confirmed in all of our numerical exercises. Taken together, Proposition 2 therefore allows us to argue that canonical models generate counterfactual implications on the rate wedge.

Our model: the private-information-based pricing effect. As illustrated by Panel A in Figure 2, the "private-information-based pricing" says that i) Bank A with a more favorable specialized signal offers a lower rate, and ii) rejects the borrower when s falls below a certain threshold x). This naturally pushes us int closer to obtaining a negative interest rate wedge.

However, the early discussion regarding "bids versus winning bids" around Figure 4 suggests that whether Bank B rejects (by quoting $r^B = \infty$) or not plays a role. As discussed, the counterfactual prediction $\Delta r > 0$ is more likely to occur if Bank B rejects more often (so Bank A enjoys a higher information rent). Hence, the private-information-based pricing effect is more likely to prevail in a positive-weak equilibrium where Bank B never rejects upon receiving a high signal and it even enjoys some "local monopoly power" as the only lender (when Bank A withdraws upon $s < x$) by having a point mass at $\bar{r}$. Note that this point mass is the distinct feature of our model with a private specialized signal compared to canonical settings a la Broecker (1990).[19] As a result, when Bank B never withdraws from the competition upon receiving $g^B = H$, the better-informed Bank

[17] Even under $\bar{r} = 36\%$, the parameters that we back out from matching three empirical moments (average loan approval rate, non-performing loan rates for both specialized and non-specialized lenders) are $q = 0.6846$, $\alpha^A = 0.9655$ and $\alpha^B = 0.9512$, implying a strictly positive interest rate wedge (0.3%). In fact, under this set of parameters (q and α's), the threshold of $\hat{\bar{r}}$ takes a value of 395%, which is significantly above the usury rate of 36%. For more details, see "Calibration" in Appendix A.3 on Page 46.

[18] We only need to verify the second condition in Part 2 of Proposition 2, which is independent of $\bar{r}$. As a brief summary, we calibrate q and α based on two empirical moments in the U.S. banking industry. First, according to this Federal Reserve report the non-performing loan (NPL) ratio is about 2%; second, Yates (2020) reports that that the approval rate for business C&I loans ranges from 55% (small firms) to 80% (large firms). Matching these two moments in Appendix A.3 we show that the implied parameters satisfy Proposition 2. For instance, taking an approval rate of 70%, we obtain $q = 0.9629$ and $\alpha = 0.716$, which satisfy $q \geq 1 - \alpha + \alpha^2$.

[19] In canonical models, although the weak bank may earn some positive profits given a high borrower surplus (say large q and $\bar{r}$), it never has a point mass at $\bar{r}$ to enjoy "local" monopoly power. To see the intuition, note that because in canonical settings information is used to determine participation, the strong lender never withdraws upon H; and since only one lender can have a point mass at $\bar{r}$ (a result that is similar to Lemma 1 for canonical models), it must indeed be the strong lender who possesses such a point mass.

A undercuts rates to win higher quality borrowers while leaving those lemons to Bank B (who then makes loans with higher winning bids).

Is $\pi^B > 0$ a necessary condition? A special case. The above discussion suggests that a profitable weak bank is necessary for a negative interest rate wedge. This is not true. The following proposition focuses on the special case of $\bar{r} = \infty$, and considers a degenerate general fundamental (so Bank B is uninformed) and a uniformly distributed specialized signal.

Proposition 3. ***(A Special Case of Uniform Distribution)*** *Suppose that $\bar{r} = \infty$ so that rejection is off equilibrium, general signals are degenerate ($q_g = 1$ or $\alpha_u = \alpha_d = 0.5$), and the specialized signal's distribution follows $\phi(s) = 1 + \epsilon\,[2 \cdot \mathbf{1}_{s \le 0.5} - 1]$. In equilibrium, i) $\pi^B = 0$ always, ii) $\Delta r = 0$ when $\epsilon = 0$ (i.e., $s \sim \mathbb{U}[0,1]$), and iii) $\Delta r > 0$ ($\Delta r < 0$) when $\epsilon > 0$ ($\epsilon < 0$) for infinitesimal ϵ.*

Several important implications of this proposition ensue. First, $\pi^B > 0$ is not necessary for $\Delta r < 0$, as we have $\pi^B = 0$ always for an uninformed Bank B. Intuitively, when $\bar{r} = \infty$, Bank B never withdraws in equilibrium regardless of its profit. As discussed after Figure 4, it is the endogenous withdrawal from the weaker bank—not profitability per se—that plays a key role in driving the difference between bids and winning bids.

Second, when the specialized signal follows a uniform distribution (together with a degenerate general signal and $\bar{r} = \infty$), the two aforementioned effects—information rent and private-information-based pricing—equalize, and lenders have the same winning bids on their granted loans. Then, starting from this benchmark, any tilting toward private-information-based pricing—e.g., tilting more probability mass toward favorable specialized signals and therefore lower rates—would generate a negative interest rate wedge observed in the data.

Model comparative statics on interest rate wedge

Figure 5 plots the comparative statics of interest rate wedge Δr with respect to model parameters, with regions of zero-weak and positive equilibria highlighted. The top two panels (A and B) concern information technology parameters α (precision of general signals) and τ (precision of the specialized signal). The overall pattern is that when information technology improves—either the general signal precision α (Panel A) or the specialized signal precision τ (Panel B)—the economy is more likely to be in the zero-weak equilibrium where the nonspecialized Bank B is sufficiently "weak" and hence makes zero profits. Note, Δr is discontinuous when π^B turns zero, as Bank B reallocates a probability mass of $1 - F^B(\bar{r}^-) > 0$ from $\bar{r}$ to ∞ (see also Panel B in Figure 3).

Therefore, an improvement in signal precision tends to weaken the non-specialized lender even further. To see the intuition, observe that i) a higher general signal precision α levels the playing field on general information and hence effectively enlarges the specialized information advantage of Bank A, and ii) a higher specialized signal precision τ directly boosts Bank A's information advantage. Since the effect of private-information-based pricing tends to dominate in a positive-weak equilibrium, a sufficiently low information technology parameter helps deliver $\Delta r < 0$.

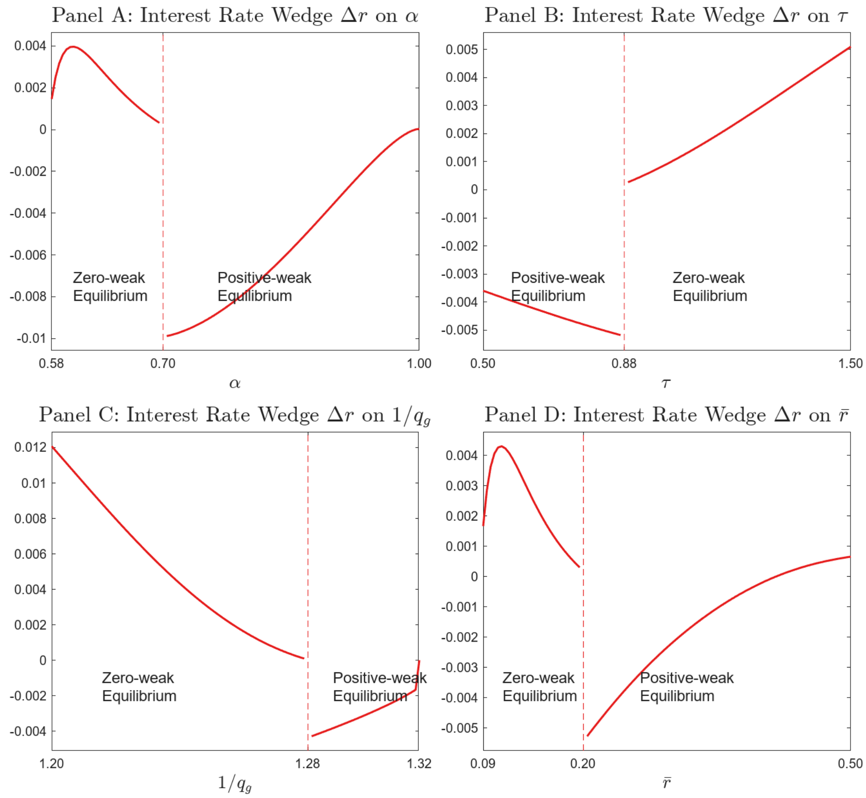

Figure 5: **Interest rate wedge.** Panel A to Panel D depict $\Delta r = \mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] - \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ as a function of α, τ, $1/q_g$ and $\bar{r}$. In Panel C, we vary $1/q_g$ but fixing the project success probability q, i.e., setting $q_s = q/q_g$. The positive-weak equilibrium arises when α or τ lies below a certain value and $1/q_g$ and $\bar{r}$ exceed a certain value. Baseline Parameters: $\bar{r} = 0.25$, $q_g = 0.8$, $q_s = 0.95$, $\tau = 0.5$ and $\alpha_u = \alpha_d = \alpha = 0.9$. Note τ captures the signal-to-noise ratio of Bank A's specialized information technology as $s = \mathbb{E}[\theta_s | \theta_s + \epsilon]$ and $\epsilon \sim \mathcal{N}(0, 1/\tau)$.

Panel C conducts another comparative static which captures the relative importance of general versus specialized information. More specifically, consider varying $1/q_g$ but fixing the project success probability q, which implies that $q_s = q/q_g$. The companion paper by He, Huang, and Parlatore (2024) explains that this comparative static exercise corresponds to the scenario in which general signals increase their span so that they cover more fundamental states critical to the success of the funded project.[20] Interestingly, this exercise yields an opposite comparative statics to the standard information technology parameters (α and τ in the top two panels) modeled as signal precision. Intuitively, now Bank B, equipped with general information that covers more fundamental states, becomes relatively stronger (rather than weaker when α and/or τ increase), so the credit market

[20] As explained in Section 2.3 where we introduce multi-dimensional fundamental states, He, Huang, and Parlatore (2024) interpret $\theta_g \equiv \prod_{n=1}^{\hat{N}} \theta_n$ ($\theta_s \equiv \prod_{n=\hat{N}+1}^{N} \theta_n$) as the borrower's "hard" ("soft") fundamental state, and model the expansion of the span of "hard" information by an increase in $\hat{N}$ (so θ_g covers more fundamental states). In the short-run, this expansion of $\hat{N}$ does not alter the span of the soft signal so that θ_g and θ_s overlap (as both have their own $\hat{N}$'s), but in the long-run the coverage of θ_s also shrinks so that θ_g and θ_s do not overlap. Panel C corresponds to the long-run scenario. For the short-run scenario, the expansion of $\hat{N}$ induces a correlation between θ_s and θ_g, which makes the analysis a bit involved but still tractable. For more details, see He, Huang, and Parlatore (2024).

equilibrium is more likely to be in the region of positive-weak (and delivers a negative interest rate wedge). Motivated by the recent advancements in big data technology, He, Huang, and Parlatore (2024) employ this framework to study the concept of "hardening soft information."

Finally, Panel D studies the rate cap $\bar{r}$ which also captures the total surplus in this economy. When the total surplus increases, the credit market equilibrium moves to the positive-weak region, which is intuitive. We observe that Δr jumps down to be negative first, then increases and turns even positive when $\bar{r}$ is sufficiently high. This is consistent with Proposition 3, in that the sign of Δr does not depend on the sign of π_B. This result highlights the robustness of our mechanism of private-information-based pricing.

Connection to the IO literature on imperfect competition and adverse selection

Our study of the interest rate wedge between asymmetrically informed lenders is related to the industrial organization (IO) literature on imperfect competition and adverse selection (see (Mahoney and Weyl, 2017; Crawford, Pavanini, and Schivardi, 2018)). Within that body of literature, market power (of lenders) and adverse selection (of borrowers) are considered distinct market frictions. Market power pertains to the situation where the demand for the firm's (differentiated) products remains relatively inelastic with respect to its price, whereas adverse selection is characterized by the observation that the effective revenue of marginal consumers decreases as the firm raises its price.[21] Piecing these two forces together, the key takeaway is an interaction effect: while firms with greater market power should charge higher prices, this standard force should be attenuated by adverse selection, which hurts marginal revenue when firms raise their prices.

We highlight two points. First, different from the IO literature which takes market power and adverse selection as two independent market frictions, our theory takes "information asymmetry" as the primitive, with winner's curse faced by asymmetrically informed lenders as the only underlying economic force. Although one could broadly link the above-mentioned market power and adverse selection to unobservable borrower types, they are different conceptually. First, strictly speaking, there is no "market power" enjoyed by the specialized lender in our model; money from any funding source is perfectly fungible just like in Huang (2023). Moreover, there is no "adverse selection" from borrowers either, because both types of borrowers will take loans at any interest rate.[22]

Second, prices in the above-mentioned IO literature are "bids" as opposed to "winning bids;" for instance, Crawford, Pavanini, and Schivardi (2018) only consider bidding prices. Section 4.1 has highlighted the importance of distinguishing bids and winning bids in the setting of credit market competition with endogenous rejection, and future research should study whether this difference

[21] In the insurance market example used in Mahoney and Weyl (2017), a higher insurance premium is associated with lower-quality insurance buyers and hence a higher service cost. In Crawford, Pavanini, and Schivardi (2018) which studies the enterprise loan market, a higher interest rate may attract worse borrowers or induce riskier projects, leading to lower interest revenues.

[22] To the point of market power, Huang (2023) studies the competition between collateral-backed bank lending (say Citibank) and revenue-based fintech lending (say Square); borrowers view each dollar the same regardless of the lender's identity. To the point of adverse selection, as typical in corporate finance literature (e.g., Tirole, 2010) we are implicitly assuming that both types of borrower receive nonpledgeable private benefits from the project, so they strictly prefer to take the loan even if $r = \bar{r}$.

Table 1: **Interest Rate and Loan Performance**

	(1)	(2)	(3)	(4)	(5)	(6)
		Interest Rate			Non-Performing Loans	
Specialized Bank	-0.076***	-0.150***	-0.082***	-0.008***	-0.005***	-0.005***
	[0.006]	[0.007]	[0.007]	[0.001]	[0.001]	[0.001]
Log Loan Amount	-0.156***	-0.170***	-0.178***	-0.000	-0.000*	-0.001**
	[0.002]	[0.002]	[0.002]	[0.000]	[0.000]	[0.000]
Constant	4.992***	5.118***	5.178***	0.045***	0.047***	0.049***
	[0.019]	[0.018]	[0.018]	[0.002]	[0.002]	[0.002]
Year-Quarter FE	X	X	X	X	X	X
Purpose FE	X	X	X	X	X	X
Rating Category (1-3) FE	X	X	X	X	X	X
Bank-Year FE		X	X		X	X
Industry-Year FE			X			X
R^2	0.31	0.39	0.4	0.031	0.044	0.047
N	353,544	353,537	351,776	353,544	353,537	351,776

Note: In Columns (1) – (3), we regress the loan rate on the fixed effects specified at the bottom of the table and a dummy denoting whether the firm is borrowing from a bank that is specialized in the industry in which said firm operates. We define a bank as specialized if it is over-invested by 4% or more in an industry, relative to what would be expected from diversification. In Columns (4) – (6), we use the same specifications as in previous columns, but make use of whether the loan in question ever becomes non-performing at any date it is in our sample after its origination. A loan becomes non-performing if it is ever in arrears, has not been paid down at maturity, or defaults outright. Standard errors are clustered at the firm-time level and are heteroskedasticity robust while *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

can reverse the conclusions from the IO literature.

4.2 Lower Rates and Better Performance: Empirical Evidence

Our model's two main testable predictions pertain to differences in loan pricing and performance between specialized and nonspecialized banks, and we have provided supporting evidence based on raw differences shown in Figure 1. In this section, we conduct a more rigorous empirical analysis of these two testable hypotheses, using supervisory data collected by the Federal Reserve System (Y14Q-H.1) which covers all C&I loans to which a stress-tested bank has committed over one million USD between 2012 and 2023. In Appendix B, we provide further details on the data, variable construction, and regression specifications.

To determine whether a bank is specialized in an industry, we look at their "excess specialization" as defined in Blickle, Parlatore, and Saunders (2023). More specifically, we say that a bank b is specialized in industry s at time t if they are "over-invested" by over 4% relative to the overall share of industry s in the overall C&I lending portfolio in our sample, i.e.

$$\frac{LoanAmount_{b,s,t}}{\sum_s LoanAmount_{b,s,t}} - \frac{LoanAmount_{s,t}}{\sum_s LoanAmount_{s,t}} \geq 4\%.$$

Under this threshold, the average bank specializes in 2.8 industries; the average overinvestment is 8.9% for specialized banks, while only 0.2% for nonspecialized ones. Our analyses below are robust to using 3% or 5% as a threshold.

Baseline results. We consider the following specification that relates our variable of interest y_{libst}, either the loan rate or performance, for a bank b's loan l to borrower i in industry s in quarter-year t, to a dummy $Specialized_{bst}$ that denotes whether the bank b in question is specialized in the industry s at time t:

$$y_{libst} = \beta_0 + \beta_1 \cdot Specialized_{bst} + \beta_2 \cdot Size_{lt} + \xi_{bt} + \sigma_{st} + \phi_{\text{rating-category}} + \omega_{\text{loan-purpose}} + \epsilon_{libst}. \quad (30)$$

The inclusion of controls and fixed effects in (30) deserves further discussion. In our model, loans are fixed-size and have the same purpose. Hence, we control for the loan's size and purpose to ensure these characteristics do not drive our findings.

Similarly, our model is conditional on firm characteristics that are observable to both lenders. Banks can partially assess the riskiness of the loan from many observable characteristics such as EBIT, ROA, and assets-to-debt. Ideally, we would control for these observables to compare loans to firms with the same "observable" riskiness. However, over 75% of the firms in our sample are private firms, for many of which we do not have data on performance. Hence, computing traditional risk metrics is difficult. To address this issue, we use banks' internal risk ratings, which contain limited inside information because they must be defensible to Federal Reserve examiners. Using these ratings, we classify the banks as high-risk, mid-risk, and safe based on the internal ratings of the loans. We add the potentially time-varying dummies $\phi_{\text{rating-category},t}$ in our regression (30) to capture the public information on the borrower/loan riskiness.

Columns (1) – (3) in Table 1 show a negative correlation between banks being specialized and their loan rates in their industry of specialization, where we subsequently introduce bank-year and industry-year fixed effects to control for any time-varying heterogeneity among banks and industries. This is the empirical counterpart to the negative interest rate wedge we studied in the previous section. Magnitude-wise, the identified negative wedge (8∼15 bps) is below the raw difference of about 40 bps shown in Figure 1, presumably due to better control in our richer specification in the regression in Eq. (30). Furthermore, Columns (4) – (6) in Table 1 show a significantly negative correlation between specialization and non-performance.[23] As one would expect if bank specialization is driven by the banks' informational advantage as modeled in this paper, specialized lenders pick higher quality loans, which are less likely to turn non-performing later.

Specialization versus competitiveness of loan market Generally speaking, the competitiveness in the loan market in an industry is an important determinant of loan prices. In our model, bank competition is quite stark: the winner takes it all in that a firm borrows from one lender only. Hence, although firm-fixed effects are usually used to control for borrower-specific time-varying factors, it is inappropriate to include them in our regression because the firms sorting into specialized

[23] Non-performing loans are those that fall into arrears, are not paid down by the end of their maturity, default or require renegotiation due to covenant violation issues. The average non-performance rate of loans throughout our sample is around 5%.

Table 2: **Interest Rate and Loan Performance: Controlling for Lending Market Competition among Specialized Banks**

	(1)	(2)	(3)	(4)	(5)	(6)
		Interest Rates			Non-Performing Loans	
Specialized Bank	-0.454***	-0.179***	-0.112***	-0.019***	-0.007	-0.007
	[0.037]	[0.036]	[0.038]	[0.005]	[0.005]	[0.005]
Log Loan Amount	-0.157***	-0.171*	-0.178**	-0.000	-0.001*	-0.001**
	[0.002]	[0.002]	[0.002]	[0.000]	[0.000]	[0.000]
Competitive Industry	-0.149***	-0.125***		-0.012***	-0.011***	
	[0.008]	[0.007]		[0.001]	[0.001]	
Interaction: Spec. Bank* Comp. Ind.	0.407***	0.047	0.032	0.012**	0.004	0.002
	[0.037]	[0.037]	[0.039]	[0.005]	[0.005]	[0.005]
Constant	5.120***	5.230***	5.178***	0.055***	0.056***	0.049***
	[0.020]	[0.020]	[0.020]	[0.002]	[0.003]	[0.002]
Year-Quarter FE	X	X	X	X	X	X
Purpose FE	X	X	X	X	X	X
Rating Category (1-3) FE	X	X	X	X	X	X
Bank-Year FE		X	X		X	X
Industry-Year FE			X			X
R^2	0.31	0.39	0.4	0.031	0.044	0.047
N	353,544	353,537	351,776	353,544	353,537	351,776

Note: In Columns (1) – (3), we regress the loan rate on the fixed effects specified at the bottom of the table and a dummy denoting whether the firm is borrowing from a bank that is specialized in the industry in which said firm operates. We define a bank as specialized if it is over-invested by 4% or more in an industry, relative to what would be expected from diversification. We interact our variable of interest with a dummy that takes the value of 1 if the industry in question has a "competitive" lending market among specialized lenders, i.e., there are more than one bank that specializes in that industry. In Columns (4) – (6), we use the same specifications as in previous columns, but with "non-performing" indicator as the dependent variable. A loan becomes non-performing if it is ever in arrears, has not been paid down at maturity, or defaults outright. Standard errors are clustered at the firm-time level and are heteroskedasticity robust while *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

and non-specialized banks is a key feature of the mechanism we highlight. To partially address the issue, we include the time-varying rating category dummies in our regression in Eq. (30) to absorb (observable) borrower-specific time-varying factors.

Additionally, competition in our model is between asymmetrically informed lenders while in practice competition may also occur among specialized banks. To ensure that our results are not driven by competition among specialized banks, Table 2 expands Table 1 with additional control for a bank being specialized in a "competitive" industry, where we define the loan market being competitive for an industry if two or more banks specialize in it. More specifically, we add the dummy "Competitive Industry," which indicates whether there is competition among specialized lenders in that industry, and the interaction between "Specialized Bank" and "Competitive Industry." Under the alternative mechanism, the specialized lender charges lower rates only because it faces fiercer competition from other specialized banks, and therefore the significantly negative effect on "Specialized Bank" in Table 1 would be fully absorbed by the interaction term in Table 2.

Columns (1) – (3) in Table 2 support the economic mechanism proposed by our model, not the alternative. We observe a negative coefficient for "Competitive Industry," potentially because industries with more specialized lenders have better quality borrowers.[24] But the coefficients of the interaction term are either positive or insignificant across all three specifications (Columns (1) – (3)), inconsistent with the alternative mechanism of competition among specialized lenders. Finally, the results on loan performance are not as robust as those on interest rates; while the point estimates in Columns (4) – (6) are negative in all our specifications, they are not statistically significant in our most saturated specifications.

4.3 Generalized Information Structure

As explained in Section 2.2, we have assumed a multiplicative setting with two independent fundamental states—the general and specialized states. This assumption has two key important features that drive the tractability of our model, and our solution techniques apply to any generalized information structure that maintains these two desirable features.

Two key properties for model tractability

Decisive general signal. In many scenarios the computer-based general information signal is usually used as pre-screening and decisive for loan granting, while the specialized signal collected by the specialized bank tailors interest rate terms (see Section 2.3). To capture this commonly observed lending practice, the multiplicative structure makes the "general" state decisive in project success, making such lending strategies more likely to arise in equilibrium.

Independence conditional on project success. Formally,

$$\tilde{g}^A \perp\!\!\!\perp \tilde{g}^B \perp\!\!\!\perp \tilde{s} \mid \theta = 1 \ . \tag{31}$$

Conditional on project success, all signals—including the specialized one by lender A and two general ones by both lenders—are independent of each other. One can easily verify that our setting in Section 2.3 with independent general and specialized states satisfies (31), although tractability does not rely on independent general and specialized states. Consider the following example studied by He, Huang, and Parlatore (2024) with $\theta = \theta_1\theta_2\theta_3$, $\theta_g = \theta_1\theta_2$ and $\theta_s = \theta_2\theta_3$. This information structure generalizes (7) in Section 2.3, while still satisfies (31).[25] Since our setting below allows the general and specialized signal to be correlated, it can be used to study credit market applications such as data sharing and credit registries that induce correlated lender signals.

[24] This hypothesis is further supported by the negative coefficients for "Competitive Industry" in columns (4) – (6), where we take the non-performing dummy as the dependent variable.

[25] The multiplicative structure in (7) is the key: $\theta = 1$ implies that all fundamental states $\{\theta_n, n \in 1, ..., N\}$ take the value of one. Unconditionally, however, the pair-wise correlations of $\{\tilde{g}^A, \tilde{g}^B, s\}$ are all positive, simply because the general state θ_g and specialized state θ_s are correlated.

Equilibrium characterization under generalized information structure

We now solve for the credit market equilibrium under a general information structure, with two major assumptions as outlined above. First, lenders only participate given an H general signal, with parameter restrictions in the same spirit as Assumption 1 but tailored for the generalized information structure; details are provided in Appendix A.5. Second, conditional on the project's state $\theta = 1$, signals are independent across general and specialized and across lenders. Since the major derivation is also available in He, Huang, and Parlatore (2024), we keep the presentation minimal here (with detailed analysis available in Appendix A.5).

Consider a specialized signal $z \sim \phi_z(z)$ for $z \in [\underline{z}, \overline{z}]$ where both $\underline{z}$ and $\overline{z}$ can be unbounded. Denote by $\mu_{g^A g^B}(z) \equiv \mathbb{P}\left(\theta = 1 \middle| g^A, g^B, z\right)$ the posterior probability density for $\theta = 1$, i.e., the state of project success. Without loss of generality, we assume that $\mu_{HH}(z)$ strictly increases in z (as we can always use $\mu_{HH}(z)$ as a signal; recall the posterior s serves as the signal in the baseline model given in Section 2). This implies that just as in the baseline, there exists $\hat{z}$ at which Bank A starts quoting $\overline{r}$, and z_x below which it starts rejecting borrowers. Let $\overline{\mu}_{g^A g^B} \equiv \mathbb{P}\left(\theta = 1 | \, g^A, g^B\right)$ denote the posterior probability of $\theta = 1$ based on general signals.

Let $p_{g^A g^B}(z) \equiv \mathbb{P}\left(g^A, g^B, z\right)$, $\overline{p}_{g^A g^B} \equiv \mathbb{P}\left(g^A, g^B\right)$, and $\alpha_u^j \equiv \mathbb{P}\left(g^j = H | \, \theta = 1\right)$ for $j \in \{A, B\}$ (so two lenders can differ in their precisions in general signals). Finally, let $\phi_z(z | \, \theta = 1)$ be the density of z conditional on $\theta = 1$. The following proposition generalizes Proposition 1.

Proposition 4. ***(Credit Market Equilibrium under General Information Structure)*** *Lender $j \in \{A, B\}$ rejects the borrower (by quoting $r = \infty$) upon $g^j = L$; when $g^j = H$, lender j may make offers from a common support $[\underline{r}, \overline{r}]$ (or reject) with the following properties.*

1. *Bank A who observes a specialized signal z offers*

$$r^A(z) = \begin{cases} \min\left\{ \frac{\pi^B + \int_{\underline{z}}^{z} p_{HH}(t)dt + \overline{p}_{LH}}{\int_{\underline{z}}^{z} p_{HH}(t)\cdot\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1, \overline{r} \right\}, & \text{for} \quad z \in [z_x, \overline{z}] \\ \infty, & \text{for} \quad z \in [\underline{z}, z_x). \end{cases} \tag{32}$$

   This equation pins down $\underline{r} = r^A(\overline{z})$, $\hat{z} = \sup\left\{z : r^A(z) = \overline{r}\right\}$, and $z_x = \sup\left\{z : r^A(z) = \infty\right\}$.

2. *Bank B makes an offer by randomizing its rate according to:*

$$F^B(r) = \begin{cases} \frac{\alpha_u^A}{\alpha_u^B}\left[1 - \int_{\underline{z}}^{z^A(r)} \phi_z(t | \, \theta = 1)\, dt\right], & \text{for} \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B = 0\}} \cdot \left\{1 - \frac{\alpha_u^A}{\alpha_u^B}\left[1 - \int_{\underline{z}}^{\hat{z}} \phi_z(t | \, \theta = 1)\, dt\right]\right\}, & \text{for} \quad r = \overline{r}. \end{cases} \tag{33}$$

3. *The endogenous non-specialized Bank B's profit $\pi^B \geq 0$ is determined similarly as Lemma 2, with detailed expression provided in Appendix A.5.*

Proposition 4 shows that the simple equilibrium structure survives under the more generalized information structure. This is because lenders only consider the marginal good type borrower who is payoff relevant, so the key argument in the baseline model still applies given signals' independence conditional on project success. As a result, the effects of specialized and general signals on equilibrium strategies are separable, and a simple characterization as in Proposition 4 ensues.

4.4 Information Acquisition and Endogenous Specialization

Although the information structure is likely to be fixed in the short run, in the long run, banks can choose what type of information—general and/or special— they want to have about borrowers. In this section, we study the lender's information acquisition problem and derive conditions under which the information structure assumed in the baseline model is an equilibrium outcome.

Setting and information acquisition technologies. We introduce another borrower firm—which we call b—in addition to the borrower firm (we call it a) in our baseline model. We may equally interpret a and b as different industries. Two types of technologies respectively relate to "general" information and "specialized" information. For the "general" information technology, a lender j invests once in equipment at a cost of κ_g, which allows the lender to process data (say financial and operating data) and produce a private *general information* signal $g_i^j \in \{H, L\}$ for each firm $i = a, b$. This captures the idea that general information is collected via standardized and transferable data such as credit reports and income statements, so once the IT equipment, software, and APIs are installed, credit analysis is easy to implement on multiple firms. As before $g_i^j \in \{H, L\}$ are independent (across two lenders and two firms) conditional on the general fundamental θ_g.

For the "specialized" information technology, a lender needs to collect specialized information on firms one by one. Lender i specializes in firm j if it spends κ_s to acquire a *specialized- information-based* private signal s_i^j, whose distribution follows the CDF $\Phi(s)$ and pdf $\phi(s)$ for $s \in [0, 1]$. If a bank wants to acquire specialized information about both firms, it needs to pay $2\kappa_s$.

We are interested in the following equilibrium: Bank A (B) endogenously specializes in firm a (b)—i.e., acquires both general and specialized signals on firm a (b)—and competes with the other non-specialized Bank B (A) who only acquires general signal on firm a (b). Given this equilibrium structure, we omit the indexation for firm i from now on when referring to the specialized signals. The baseline model analyzed in Section 3 is the subgame for either firm following the equilibrium information acquisition strategies.

Incentive compatibility conditions. Banks make their information acquisition decisions simultaneously, and we assume that information acquisition is observable when banks enter the credit market competition game. Therefore a lender's deviation from the proposed equilibrium information acquisition will lead to a different information structure in the credit market competition, and we need to derive equilibrium lending profits in all possible subgames following a deviation.

Denote by $\Pi_j^i\left(I_A^g, I_A^s, I_B^s, I_B^s\right)$ the expected lending profits of bank j in firm i when the information structure in firm i is given by $(I_A^g, I_A^s, I_B^g, I_B^s,)$, where I_j^g and I_j^s take value of one if bank j acquired general and specialized signals in firm i, respectively, and zero otherwise. The symmetry on industries implies that a bank's expected lending profits in firm i only depend on the information structure in that industry but not on the industry itself, i.e.,

$$\Pi_j^a\left(I_A^g, I_A^s, I_B^s, I_B^g\right) = \Pi_j^b\left(I_A^g, I_A^s, I_B^g, I_B^s\right). \tag{34}$$

Therefore, we drop index i from the expected lending profits. Moreover, we focus on Bank A's incentives in what follows since the no deviation conditions for banks A and B are symmetric.

Bank A can deviate along three dimensions: it can choose not to acquire general information, it can choose not to acquire specialized information about firm a, and it can choose to acquire specialized information in firm b. Bank A's incentives to deviate along these dimensions will depend on the costs of acquiring information. As one would expect, the lower the cost of acquiring general information, the more likely Bank A has incentives to acquire general information and not deviate along this dimension. For deviations along the specialized information dimension, the cost of acquiring specialized information has to be low enough such that it is worth acquiring specialized information in firm a and having an informational advantage over Bank B in this firm but high enough such that it is not worth acquiring specialized information in firm b to stop being the less informed lender. This intuition can be formally stated in the following incentive compatibility constraints. Bank A does not want to deviate by

1. not acquiring general information:

$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right) - \Pi_A\left(I_A^g=0, I_A^s=1, I_B^g=1, I_B^s=0\right) +$$
$$\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right) - \Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right) \geq \kappa_g; \tag{G}$$

2. not acquiring general information nor specialized information in firm a:

$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right) - \Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=0\right) +$$
$$\Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right) - \Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right) \geq \kappa_g + \kappa_s; \tag{NI}$$

3. not acquiring specialized information in firm a:

$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=0\right) - \Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=0\right) \geq \kappa_s; \tag{Sa}$$

4. and, acquiring specialized information in firm b:

$$\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=1\right) - \Pi_A\left(I_A^g=1, I_A^s=0, I_B^g=1, I_B^s=1\right) \leq \kappa_s. \tag{NSb}$$

Essentially, constraints (G) and (NI) impose an upper bound on κ_g so that Bank A wants to acquire general information. Analogously, constraints (NI) and (Sa) impose an upper bound on κ_s so that Bank A wants to acquire specialized information in firm a, while Constraint (NSb) imposes a lower bound on κ_s to ensure it does not want to be specialized in firm b.

Deviation payoffs. We aim to show that there exist cost parameters κ_g and κ_s such that the conditions above hold for some parameterization. Thus, we need to characterize the deviation payoffs. The expressions for $\Pi_A\left(I_A^g, I_A^s, I_B^g, I_B^s\right)$ are in Appendix A.6. One noteworthy point is that $\Pi_A\left(I_A^g=1, I_A^s=1, I_B^g=1, I_B^s=1\right)$ corresponds to a generalization of Riordan (1993), where each (symmetric) lender has a continuous special signal and also a binary general signal.

Parameter conditions for equilibrium with specialized lending. In our setting an uninformed bank always makes zero equilibrium profits (Milgrom and Weber, 1982):

$$\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=0\right)=\Pi_A\left(I_A^g=0, I_A^s=0, I_B^g=1, I_B^s=1\right)=0.$$

It then follows that Constraint (NI) is equivalent to the participation constraint of Bank A. Moreover, this condition implies that for any acquisition cost of specialized signal κ_s such that (Sa) is satisfied, we can always find a cost of general information κ_g small enough to satisfy (G) and (NI). Therefore an equilibrium with lending specialization emerges as long as κ_s satisfies the bounds imposed by (Sa) and (NSb). Intuitively, we need that the benefits from acquiring specialized information to become the more informed lender (e.g., getting s_A^a for Bank A, which is part of the equilibrium strategy in the baseline) are greater than the benefits from acquiring specialized information to stop being the less informed lender (e.g., getting s_A^b for Bank A which deviates from our equilibrium in the baseline). This is confirmed in Figure 6 in Appendix A.6, which depicts the range of information acquisition costs κ_g and κ_s so that the conjectured information structure with a specialized lender and the ensuring lending competition indeed form an equilibrium.

5 Concluding Remarks

One of banks' main roles in the economy is producing information to allocate credit. In this paper, we show that the nature of information produced by banks affects the credit market equilibrium and the degree of competition among banks. By considering specialized and general information, we can explain empirical patterns in bank lending specialization—the negative interest rate wedge—that are unexplained by canonical models where information technology is solely characterized by the signal's precision. In a companion paper with a similar credit market competition setting, He, Huang, and Parlatore (2024) distinguishes between the quality (signal precision) and breadth (information span) of information, a distinction that is crucial to understanding the changing landscape in the credit market due to technological advances related to data gathering and processing that lead to the hardening of soft information.

From a modeling perspective, including a continuously distributed signal within a credit market equilibrium enables us to examine private-information-based pricing, a pertinent aspect of significant importance in the banking sector in practice. Furthermore, by incorporating both specialized and general information—which reflect potentially many more underlying states—among asymmetric lenders, our paper markedly advances the field of common-value auction literature involving such asymmetrically informed lenders in which each lender possesses private information (in contrast to Milgrom and Weber (1982) where one bidder knows strictly more than the other). We fully characterize the equilibrium in closed form and anticipate broader applications based on our framework and solution methodology.

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A Technical Appendices

A.1 Credit Competition Equilibrium

Proof of Lemma 1

Proof. Note that the property of no gaps implies common support $[\underline{r}, \overline{r}]$. This is because, if a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this is one example of gaps in the first bank's support.

To show that the distributions have no gap, suppose that, say, the support of Bank B's interest rate offering F^B has a gap $(r_1, r_2) \subset [\underline{r}, \overline{r}]$.Then F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. It follows that at least one lender, whose competitor's interest rate offering does not have a mass point at r_1 (it is impossible that both distributions have a mass point at r_1), has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$. Contradiction.

Regarding point mass, suppose that one distribution, say F^B has a mass point at $\tilde{r} \in [\underline{r}, \overline{r})$. Then Bank A would not quote any $r^A(s) \in [\tilde{r}, \tilde{r} + \epsilon]$ and it would strictly prefer quoting $r^A = \tilde{r} - \epsilon$ instead. In other words, the support of F^A must have a gap in the interval $[\tilde{r}, \tilde{r} + \epsilon]$. This contradicts the property of no gaps which we have shown. Finally, it is impossible that both distributions have a mass point at $\overline{r}$.

□

A.2 Proof of Proposition 1

Proof. This part proves that Bank A's equilibrium interest rate quoting strategy as a function of specialized signal $r^A(s)$ is always decreasing; this implies that the FOC that helps us derive Bank A's strategy also ensures the global optimality.

Write Bank A's value $\Pi^A(r, s)$ as a function of its interest rate quote and specialized signal, in the event of $g^A = H$ and s. (We use π to denote the equilibrium profit but Π for any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r, s) = \underbrace{p_{HH}}_{g^A=H, g^B=H} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HH} s (1+r) - 1\right] + \underbrace{p_{HL}}_{g^A=H, g^B=L} \left[\mu_{HL} s (1+r) - 1\right] \tag{35}$$

with the following FOC:

$$0 = \underbrace{p_{HH} \left[-\frac{dF^B(r)}{dr}\right]}_{\text{lost customer}} \left[\underbrace{\mu_{HH} s (1+r) - 1}_{\text{customer return}}\right] + \underbrace{p_{HH} \left[1 - F^B(r)\right]}_{\text{customer}} \underbrace{\mu_{HH} s}_{\text{MB of customer}} + p_{HL} \mu_{HL} s. \tag{36}$$

One useful observation is that on the support, it must hold that $\mu_{HH} s (1+r) - 1 > 0$; otherwise, $\mu_{HL} s (1+r) - 1 < \mu_{HH} s (1+r) - 1 \leq 0$, implying that Bank A's profit is negative (so it will exit).

Lemma 3. *Consider s_1, s_2 in the interior domain with corresponding interest rate quote r_1 and r_2. The marginal value of quoting r_2 for type $s = s_1$ is*

$$\Pi_r^A(r_2, s_1) = \frac{s_2 - s_1}{\mu_{HH} s_2 (1+r_2) - 1} \left\{p_{HH} \left[1 - F^B(r_2)\right] \mu_{HH} + p_{HL} \mu_{HL}\right\}$$

and its sign depends on the sign of $s_2 - s_1$.

Proof. Evaluating the FOC (36) of type s_1 but quoting r_2:

$$\Pi_r^A(r_2, s_1) = p_{HH} \left[-\frac{dF^B(r_2)}{dr}\right] \left[\mu_{HH} s_1 (1+r_2) - 1\right] + p_{HH} \left[1 - F^B(r_2)\right] \mu_{HH} s_1 + p_{HL} \mu_{HL} s_1. \tag{37}$$

FOC at type s_2 yields

$$\Pi_r^A(r_2,s_2) = p_{HH}\left[-\frac{dF^B(r_2)}{dr}\right]\left[\mu_{HH}s_2(1+r_2)-1\right] + p_{HH}\left[1-F^B(r_2)\right]\mu_{HH}s_2 + p_{HL}\mu_{HL}s_2 = 0,$$

or

$$\frac{dF^B(r_2)}{dr} = \frac{p_{HH}\left[1-F^B(r_2)\right]\mu_{HH}s_2 + p_{HL}\mu_{HL}s_2}{p_{HH}\left[\mu_{HH}s_2(1+r_2)-1\right]}. \tag{38}$$

Plugging in this term to (37), $\Pi_r^A(r_2,s_1)$ becomes

$$\begin{aligned}
&-\frac{\mu_{HH}s_1(1+r_2)-1}{\mu_{HH}s_2(1+r_2)-1}\left\{p_{HH}\left[1-F^B(r_2)\right]\mu_{HH}s_2 + p_{HL}\mu_{HL}s_2\right\} + p_{HH}\left[1-F^B(r_2)\right]\mu_{HH}s_1 + p_{HL}\mu_{HL}s_1\\
&=\left[s_1 - \frac{\mu_{HH}s_1(1+r_2)-1}{\mu_{HH}s_2(1+r_2)-1}\cdot s_2\right]\left\{p_{HH}\left[1-F^B(r_2)\right]\mu_{HH} + p_{HL}\mu_{HL}\right\}\\
&=(s_2-s_1)\cdot\frac{p_{HH}\left[1-F^B(r_2)\right]\mu_{HH} + p_{HL}\mu_{HL}}{\mu_{HH}s_2(1+r_2)-1},
\end{aligned}$$

which is the claimed marginal benefit of quoting r_2 for type s_1. Its sign depends on $s_2 - s_1$ because the denominator is positive as we noted right after Eq. (36). □

Lemma 3 has three implications. First, as long as $r^A(\cdot)$ is (strictly) increasing in some segment, then Bank A would like to deviate in this segment. To see this, suppose that $r_1 > r_2$ when $s_1 > s_2$ for s_1, s_2 arbitrarily close. Because Lemma 1 has shown that Bank A's strategy is smooth, r_2 is arbitrarily close to r_1. Then $\Pi_r^A(r_2,s_1) < 0$, implying that the value is convex and the Bank A at s_1 (who in equilibrium is supposed to quote r_1) would like to deviate further.

Second, the monotonicity implied by Lemma 3 helps us show that Bank A uses a pure strategy. To see this, for any $\hat{s} \geq s_1 > s_2$ that induce interior quotes $r_1, r_2 \in [\underline{r}, \overline{r})$, however close, in equilibrium we must have $\sup r^A(s_1) < \inf r^A(s_2)$ by monotonicity. Combining this with Part 3 of Lemma 1, i.e., the induced distribution $F^A(\cdot)$ is atomless except for at $\overline{r}$ and has no gaps, we know that Bank A must adopt a pure strategy in the interior of $[\underline{r}, \overline{r})$, or for $s \leq \hat{s}$. Finally, on $s < \hat{s}$ Bank A can quote either $\overline{r}$ or ∞ which generically gives different values; this then rules out randomization.

Third, if $r^A(\cdot)$ is decreasing globally over $\mathcal{S}$, then the FOC is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r} > r_1$; then

$$\Pi^A(\check{r},s_1) - \Pi^A(r_1,s_1) = \int_{r_1}^{\check{r}} V_r^A(r,s_1)\,dr.$$

Given the monotonicity of $r(s)$, we can find the corresponding type $s(r)$ for $r \in [r_1, \check{r}]$. From Lemma 3 we know that

$$\Pi_r^A(r,s_1) = (s(r)-s_1)\frac{p_{HH}\left[1-F^B(r)\right]\mu_{HH} + p_{HL}\mu_{HL}}{\mu_{HH}s(r)(1+r)-1},$$

which is negative given $s(r) < s_1$. Therefore the deviation gain is negative. Similarly, we can show a negative deviation gain for any $\check{r} < r_1$.

Next, we show that $r^A(\cdot)$ is single-peaked over the space of $[0,1]$.

Lemma 4. *Given any exogenous $\pi^B \geq 0$, $r^A(\cdot)$ single-peaked over $[0,1]$ with a maximum point.*

Proof. When $r \in [\underline{r}, \overline{r})$, the derivative of $r^A(s)$ in Eq. (13) with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}\phi(s)\left(\overbrace{p_{HH}\mu_{HH}\left[\int_0^s t\phi(t)\,dt - s\Phi(s)\right]}^{M_1(s)<0,\text{ and }M_1'(s)<0} + \overbrace{p_{LH}\mu_{LH}q_s - \left(\pi^B + p_{LH}\right)\mu_{HH}s}^{M_2(s)?0,\text{ but }M_2'(s)<0}\right)}{\left(p_{HH}\mu_{HH}\int_0^s t\phi(t)\,dt + p_{LH}\mu_{LH}q_s\right)^2}.$$

As $\int_0^s t\phi(t)\,dt < s\Phi(s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -p_{HH}\mu_{HH}\Phi(s) < 0.$$

For $M_2(s) = p_{LH}\mu_{LH}q_s - \left(\pi^B + p_{LH}\right)\mu_{HH}s$, it has an ambiguous sign, but is decreasing in s. This implies that $M_1(s) + M_2(s)$ decreases with s. It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore $r^A(s)$ first increases and then decreases, i.e. single-peaked. □

Suppose that the peak point is $\tilde{s}$; then Bank A should simply charge $r(s) = \tilde{r}$ for $s < \tilde{s}$ for better profit. This is the standard "ironing" technique and we therefore define the following ironed strategy formally (here, we also take care of the capping $r \le \overline{r}$):

$$r^A_{ironed}(s) \equiv \sup_{t\in[s,1]} \min\left(r^A(t), \overline{r}\right).$$

By definition $r^A_{ironed}(s)$ is monotonely decreasing.

We now argue that in equilibrium, π^B and $\underline{r}$ adjust so that $r^A(\cdot)$ is always monotonely decreasing over $[x,1]$. (Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0,1]$ immediately follows.) There are two subcases to consider.

1. Suppose that $\tilde{r} = \overline{r}$. In this case, $r^A(s)$ in Eq. (13) used in Lemma 3 and 4 does not apply to $s < \tilde{s}$ because the equation is defined only over the left-closed-right-open interval $[\underline{r}, \overline{r})$. Instead, $r^A(s)$ in this region is determined by Bank A's optimality condition: as r^A does not affect the competition from Bank B (which equals $F^B\left(\overline{r}^-\right)$), Bank A simply sets the maximum possible rate $r^A(r) = \overline{r}$ unless it becomes unprofitable (for $s < x$). (This is our zero-weak equilibrium with $\pi^B = 0$, and there is no "ironing" in this case.)
2. Suppose that $\tilde{r} < \overline{r}$; then bank A quotes $\tilde{r}$ for all $s < \hat{s}$. But this is not an equilibrium—Bank A now leaves a gap in the interval $[\tilde{r}, \overline{r}]$, contradicting with Lemma 1 (there, we rule out gaps in equilibrium). Intuitively, Bank A is too aggressive, and Bank B always would like to raise its quotes inside $[\tilde{r}, \overline{r}]$ to $\overline{r}$. In equilibrium, π^B and $\underline{r}$ adjust upward, so that the peak point $\tilde{s}$ coincides with $\overline{r}$, resulting in no "ironing" in this case either. (This is our positive-weak equilibrium with $\pi^B > 0$.)

□

Proof of Lemma 2 We explain the logic of the proof here. Note that s_B^{be} is the highest specialized signal under which Bank A's offer hits $\overline{r}$, given $\pi^B = 0$. Moreover, recall that s_A^{be} is the level of the specialized signal under which Bank A just breaks even when quoting $\overline{r}$. If $s_B^{be} < s_A^{be}$, then we know s hits s_A^{be} (i.e., Bank A hits zero profit) first when s goes down from the top, implying that Bank A will lose money upon at $s = s_B^{be} < s_A^{be}$. Combining these two pieces, we know that quoting $\overline{r}$ at s_B^{be}—which is under the implicit assumption of $\pi^B = 0$—must be off-equilibrium for Bank A. Therefore in equilibrium $\pi^B > 0$ and Bank A withdraws itself upon $s < x = \hat{s} = s_A^{be}$. If on the other hand $s_B^{be} \ge s_A^{be}$, we are in the alternative scenario where $\hat{s} = s_B^{be}$ and $\pi^B = 0$; Bank A who is making a positive profit at s_B^{be} will keep quoting $\overline{r}$ for $s < s_B^{be}$, until $s < x$ upon which it exits.

Proof. First, we argue that equilibrium $\hat{s} \equiv \arg\sup_s \left\{s : r^A(s) \ge \overline{r}\right\}$ either equals s_A^{be} or s_B^{be}. To see this, if $\pi^B = 0$, we have $\hat{s} = s_B^{be}$ by construction. If $\pi^B > 0$, then Bank B always makes an offer upon H, i.e., $F^B(\overline{r}) = 1$. We also know that $F^B\left(\overline{r}^-\right) = 1 - \frac{\int_0^{s^A(r)=\hat{s}^+} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and so $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at $\overline{r}$. It follows that $F^A(r)$ is open at $\overline{r}$: $\hat{s} = x$ and $\pi^A\left(r^A(\hat{s})\,|\hat{s}\right) = 0$, which is exactly the definition of s_A^{be}, so $\hat{s} = s_A^{be}$ in this case.

Now we prove the claim in this lemma. In the first case of $s_B^{be} < s_A^{be}$, we have $\hat{s} \le s_A^{be}$ and thus Bank A's probability of winning when quoting $r^A = \overline{r}$ is at most $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s} \ge \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} = 1 - F^B\left(\overline{r}^-\right)$. The definition of s_A^{be} says that Bank A upon s_A^{be} breaks even when quoting $r^A\left(s_A^{be}\right) = \overline{r}$ and facing this most favorable winning probability $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s}$. Then upon a worse specialized signal $s_B^{be} < s_A^{be}$, Bank A must reject the borrower because offering $\overline{r}$ leads to losses, which rules out $\hat{s} = s_B^{be}$. According to our earlier observation of $\hat{s} = s_B^{be}$ or s_A^{be}, we have $\hat{s} = s_A^{be}$ and $\pi^B > 0$ in this case, where π^B could be characterized from Eq. (12) at $r = \overline{r}$.

In the second case of $s_B^{be} \geq s_A^{be}$, we have $\hat{s} \leq s_B^{be}$ and thus Bank B's probability of winning when quoting $r^B = \overline{r}$ is at most $\Phi\left(s_B^{be}\right) \geq \Phi\left(s\right) = 1 - F^A\left(\overline{r}^-\right)$. The definition of s_B^{be} says that Bank B breaks even when quoting $r^B = \overline{r}$ and facing this most favorable winning probability $\Phi\left(s_B^{be}\right)$. Then if the competition from A were more aggressive, say $1 - F^A\left(\overline{r}^-\right) = \Phi\left(s_A^{be}\right)$, Bank B would make a loss when quoting $\overline{r}$, so $\hat{s} = s_A^{be}$ cannot support an equilibrium. Hence, in this case, $\hat{s} = s_B^{be}$ and $\pi^B = 0$. In addition,

$$
\begin{aligned}
0 =& \frac{p_{HH} \int_0^{s_A^{be}} t\phi\left(t\right)dt}{q_s}\left[\mu_{HH} s_A^{be}\left(1+\overline{r}\right) - 1\right] + p_{HL}\left[\mu_{HL} s_A^{be}\left(1+\overline{r}\right) - 1\right] \\
=& \frac{p_{HH} \int_0^{s_B^{be}} t\phi\left(t\right)dt}{q_s}\left[\mu_{HH} x\left(1+\overline{r}\right) - 1\right] + p_{HL}\left[\mu_{HL} x\left(1+\overline{r}\right) - 1\right] \\
\geq& \frac{p_{HH} \int_0^{s_A^{be}} t\phi\left(t\right)dt}{q_s}\left[\mu_{HH} x\left(1+\overline{r}\right) - 1\right] + p_{HL}\left[\mu_{HL} x\left(1+\overline{r}\right) - 1\right],
\end{aligned}
$$

where the first equality is the definition of s_A^{be}, the second equality is Bank A's equilibrium break-even condition $0 = \pi^A\left(\overline{r}|x\right)$, and the last inequality uses $s_B^{be} \geq s_A^{be}$ in this case. Hence, $x \leq s_A^{be} \leq s_B^{be} = \hat{s}$. □

A.3 Proof of Proposition 2 and Calibration

This part studies canonical models where each lender has a (general) binary signal g^j for $j \in \{A, B\}$,

$$\mathbb{P}(g^j = H|\theta = 1) = \alpha_u^j, \quad \mathbb{P}(g^j = L|\theta = 0) = \alpha_d^j.$$

$F^j(r)$ with $j \in \{A, B\}$ indicates the distribution of lender j's interest rate offering.

Lemma 5. *For any $r \in [\underline{r}, \overline{r})$, we have*

$$\frac{F^B\left(r\right)}{F^A\left(r\right)} = \frac{\alpha_u^A}{\alpha_u^B}, \; \frac{dF^B\left(r\right)/dr}{dF^A\left(r\right)/dr} = \frac{\alpha_u^A}{\alpha_u^B}.$$

Proof. For any $r \in [\underline{r}, \overline{r})$, lenders' profit functions are

$$\pi^A = \underbrace{p_{HH}}_{B\text{ gets H}} \underbrace{\left(1 - F^B\left(r\right)\right)}_{\text{wins}}\left[\mu_{HH}\left(r+1\right) - 1\right] + \underbrace{p_{HL}}_{B\text{ gets L}}\left[\mu_{HL}\left(r+1\right) - 1\right], \tag{39}$$

$$\pi^B = \underbrace{p_{HH}}_{A\text{ gets H}} \underbrace{\left(1 - F^A\left(r\right)\right)}_{\text{wins}}\left[\mu_{HH}\left(r+1\right) - 1\right] + \underbrace{p_{LH}}_{A\text{ gets L}}\left[\mu_{LH}\left(r+1\right) - 1\right]. \tag{40}$$

These two equations imply that

$$\frac{F^B\left(r\right)}{F^A\left(r\right)} = \frac{p_{HH}\left[\mu_{HH}\left(r+1\right) - 1\right] + p_{HL}\left[\mu_{HL}\left(r+1\right) - 1\right] - \pi^A}{p_{HH}\left[\mu_{HH}\left(r+1\right) - 1\right] + p_{LH}\left[\mu_{LH}\left(r+1\right) - 1\right] - \pi^B}. \tag{41}$$

And, evaluating Eq. (39), (40) at $r = \underline{r}$ and using $F^A\left(\underline{r}\right) = F^B\left(\underline{r}\right) = 1$ gives lenders' profits:

$$
\begin{aligned}
\pi^A\left(\underline{r}\right) &= p_{HH}\left[\mu_{HH}\left(\underline{r}+1\right) - 1\right] + p_{HL}\left[\mu_{HL}\left(\underline{r}+1\right) - 1\right], \\
\pi^B\left(\underline{r}\right) &= p_{HH}\left[\mu_{HH}\left(\underline{r}+1\right) - 1\right] + p_{LH}\left[\mu_{LH}\left(\underline{r}+1\right) - 1\right].
\end{aligned}
$$

Using these in Eq. (41), we have

$$\frac{F^B(r)}{F^A(r)} = \frac{(p_{HH}\mu_{HH} + p_{HL}\mu_{HL})(r - \underline{r})}{(p_{HH}\mu_{HH} + p_{LH}\mu_{LH})(r - \underline{r})} = \frac{\mathbb{P}\left(g^A = H, \theta = 1\right)}{\mathbb{P}\left(g^B = H, \theta = 1\right)} = \frac{\alpha_u^A}{\alpha_u^B}.$$

Here, $F^B(r) = \frac{\alpha_u^A}{\alpha_u^B} F^A(r)$ immediately implies that $\frac{dF^B(r)/dr}{dF^A(r)/dr} = \frac{\alpha_u^A}{\alpha_u^B}$. □

Proof of Proposition 2

Part 1: Bad-news information structure. This structure corresponds to

$$\alpha_u^A = \alpha_u^B = 1, \quad 1 > \alpha_d^A > \alpha_d^B > 0;$$

i.e., lenders only make Type II mistakes. In this part, we use $\alpha^j \equiv \alpha_d^j$ as a lender's signal precision, which captures the probability that bad-type borrowers are correctly identified as L, and $\alpha^A > \alpha^B$.

Proof. From Lemma 5, lender bidding strategies $F^A(\cdot), F^B(\cdot)$ over $[0, \overline{r}] \cup \{\infty\}$ satisfy

$$F^B(r) = \begin{cases} F^A(r), & r \in [0, \overline{r}), \\ F^A(r^-), & r = \overline{r}. \end{cases}$$

We use this result to express Δr as a function of $F^B(r)$. Specifically,

$$\begin{aligned}
\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] &= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] r dF^A(r) + p_{HL}\int_{\underline{r}}^{\overline{r}} r dF^A(r)}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] dF^A(r) + p_{HL}} \\
&= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] r dF^B(r) + p_{HH}\overline{r}\left[1 - F^B(\overline{r})\right]^2 + p_{HL}\left[\overline{r} - \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr\right]}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] dF^B(r) + p_{HH}\left[1 - F^B(\overline{r})\right]^2 + p_{HL}} \\
&= \overline{r} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2} - \frac{\left[1 - F^B(r)\right]^2}{2}\right\} dr + p_{HL}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{-\frac{\left[1 - F^B(\overline{r})\right]^2}{2} + \frac{1}{2}\right\} + p_{HH}\left[1 - F^B(\overline{r})\right]^2 + p_{HL}},
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right] &= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^A(r)\right] r dF^B(r) + p_{LH}\int_{\underline{r}}^{\overline{r}} r dF^B(r)}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] dF^B(r) + p_{LH}F^B(\overline{r})} \\
&= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] r dF^B(r) + p_{LH}\left[\overline{r}F^B(\overline{r}) - \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr\right]}{p_{HH}\int_{\underline{r}}^{\overline{r}}\left[1 - F^B(r)\right] dF^B(r) + p_{LH}F^B(\overline{r})} \\
&= \overline{r} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2} - \frac{\left[1 - F^B(r)\right]^2}{2}\right\} dr + p_{LH}\int_{\underline{r}}^{\overline{r}} F^B(r)\, dr}{p_{HH}\left\{\frac{1}{2} - \frac{\left[1 - F^B(r)\right]^2}{2}\right\} + p_{LH}F^B(\overline{r})}.
\end{aligned}$$

Hence,

$$\Delta r \equiv \mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] - \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$$
$$= \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\}dr + p_{LH}\int_{\underline{r}}^{\overline{r}}F^B(r)\,dr}{p_{HH}\left\{\frac{1}{2}-\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\}+p_{LH}F^B(\overline{r})} - \frac{p_{HH}\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\}dr + p_{HL}\int_{\underline{r}}^{\overline{r}}F^B(r)\,dr}{p_{HH}\left\{-\frac{\left[1-F^B(\overline{r})\right]^2}{2}+\frac{1}{2}\right\}+p_{HH}\left[1-F^B(\overline{r})\right]^2+p_{HL}}. \tag{42}$$

Now we plug in the expressions of $F^B(r)$ to show that the canonical model leads to counterfactual predictions when $\overline{r}$ is relatively small. From He, Huang, and Zhou (2023),

$$F^B(r) = \frac{r-\underline{r}}{r-\underline{r}\left(1-\alpha^A\right)},$$

and the key terms are accordingly

$$\int_{\underline{r}}^{\overline{r}} F^B(r)\,dr = \overline{r}-\underline{r}-\alpha^A\underline{r}\ln\left(\frac{\overline{r}}{\underline{r}}-1+\alpha^A\right)+\alpha^A\underline{r}\ln\alpha^A,$$
$$\int_{\underline{r}}^{\overline{r}}\left\{\frac{1}{2}-\frac{\left[1-F^B(r)\right]^2}{2}\right\}dr = \frac{\underline{r}}{2}\cdot\frac{\left(\frac{\overline{r}}{\underline{r}}-1\right)^2}{\frac{\overline{r}}{\underline{r}}-1+\alpha^A}.$$

Let $M(\overline{r}) \equiv \frac{\overline{r}}{\underline{r}}-\left(1-\alpha^A\right)$. Multiply Δr by both denominators in Eq. (42) (which are positive as the probability of lending), and one can show that

$$\Delta r \propto p_{HH}\cdot\frac{\underline{r}\alpha^A}{2}\cdot\left(\frac{M-\alpha^A}{M}\right)^2\left(\frac{p_{HH}\alpha^A}{M}+p_{LH}\right)+\frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}}F^B(r)\right](p_{LH}+p_{HL})\left(\frac{\alpha^A}{M}\right)^2$$
$$+p_{LH}p_{HL}\frac{\alpha^A}{M}\left[\int_{\underline{r}}^{\overline{r}}F^B(r)\right]+(p_{HL}-p_{LH})\frac{p_{HH}}{2}\cdot\underline{r}\cdot\frac{\left(M-\alpha^A\right)^2}{M}-(p_{HL}-p_{LH})\frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}}F^B(r)\,dr\right].$$

Note that only the last term $-(p_{HL}-p_{LH})\frac{p_{HH}}{2}\left[\int_{\underline{r}}^{\overline{r}}F^B(r)\,dr\right]$ is negative. In addition, this term approaches zero as $\overline{r}\to\underline{r}=\frac{(1-q)\left(1-\alpha^B\right)}{q}$, and

$$\frac{\partial\left[\int_{\underline{r}}^{\overline{r}}F^B(r)\,dr\right]}{\partial\overline{r}} = 1-\frac{\alpha^A}{M} > 0.$$

Therefore, there exists some threshold $\hat{\overline{r}}$ such that when $\overline{r}\leq\hat{\overline{r}}$, the canonical model has counterfactual prediction $\Delta r > 0$.

□

Part 2: Symmetric information structure. This structure corresponds to

$$\alpha^j \equiv \alpha_u^j = \alpha_d^j \in \left(\frac{1}{2},1\right], \quad \text{for} \quad j\in\{A,B\}.$$

In this context, the specialized lender Bank A's signal is more precise, $\alpha^A > \alpha^B$.

Lemma 6. $\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right]$ *is equivalent to the following inequality*

$$\frac{\mathbb{P}\left(x^A = H\right) \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr + p_{HH} \int_{\underline{r}}^{\overline{r}} F^A(r)\, r dF^B(r) - p_{HH} \overline{r} \frac{\alpha^B}{2\alpha^A} \left(F^B(\overline{r})\right)^2}{p_{HH}\left[1 - F^B(\overline{r}) + \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2\right] + p_{HL}}$$
$$\leq \frac{\mathbb{P}\left(x^B = H\right) \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr + p_{HH} \int_{\underline{r}}^{\overline{r}} F^A(r)\, r dF^B(r) - p_{HH} \frac{\alpha^B}{2\alpha^A} \left(F^B(\overline{r})\right)^2 \overline{r}}{p_{HH}\left[F^B(\overline{r}) - \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2\right] + p_{LH} F^B(\overline{r})}.$$

Proof. The expected rate of a lender's loan is

$$\mathbb{E}\left[r^A \middle| r^A < r^B \leq \infty\right] \triangleq \frac{\underbrace{p_{HH}}_{\text{B gets H}} \int_{\underline{r}}^{\overline{r}} \underbrace{\left[1 - F^B(r)\right]}_{\text{A wins}} r dF^A(r) + \underbrace{p_{HL}}_{\text{B gets L}} \int_{\underline{r}}^{\overline{r}} r dF^A(r)}{p_{HH} \int_{\underline{r}}^{\overline{r}} \left[1 - F^B(r)\right] dF^A(r) + p_{HL}}, \tag{43}$$

$$\mathbb{E}\left[r^B \middle| r^B < r^A \leq \infty\right] \triangleq \frac{\underbrace{p_{HH}}_{\text{A gets H}} \int_{\underline{r}}^{\overline{r}} \underbrace{\left[1 - F^A(r)\right]}_{\text{B wins}} r dF^B(r) + \underbrace{p_{LH}}_{\text{A gets L}} \int_{\underline{r}}^{\overline{r}} r dF^B(r)}{p_{HH} \int_{\underline{r}}^{\overline{r}} \left[1 - F^A(r)\right] dF^B(r) + p_{LH} F^B(\overline{r})}. \tag{44}$$

In the first step, we rewrite the equations as functions of $dF^B(r)$ and dr which are continuous at $\overline{r}$. Using integration by parts and Lemma 5, we have

$$\int_{\underline{r}}^{\overline{r}} r dF^A(r) = rF^A(r)\Big|_{\underline{r}}^{\overline{r}} - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dr = \overline{r} - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dr = \overline{r} - \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\overline{r}} F^B(r)\, dr.$$

In the last step, although Lemma 5 does not apply at $r = \overline{r}$, it is of zero measure. Similarly, the probability of Bank A winning in competition is

$$\begin{aligned}
\int_{\underline{r}}^{\overline{r}} \left[1 - F^B(r)\right] dF^A(r) &= \int_{\underline{r}}^{\overline{r}} dF^A(r) - \int_{\underline{r}}^{\overline{r}} F^B(r)\, dF^A(r) \\
\underbrace{=}_{\text{integration by parts}} & \ 1 - \left[F^B(\overline{r}) - \int_{\underline{r}}^{\overline{r}} F^A(r)\, dF^B(r)\right] \\
\underbrace{=}_{F^A = \frac{\alpha^B}{\alpha^A} F^B} & \ 1 - F^B(\overline{r}) + \int_{\underline{r}}^{\overline{r}} \frac{\alpha^B}{\alpha^A} F^B(r)\, dF^B(r) \\
&= 1 - F^B(\overline{r}) + \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2,
\end{aligned}$$

and thus the probability of Bank B winning is the residual

$$\int_{\underline{r}}^{\overline{r}} \left[1 - F^A(r)\right] dF^B(r) = F^B(\overline{r}) - \frac{\alpha^B}{2\alpha^A}\left(F^B(\overline{r})\right)^2.$$

Similarly,

$$\begin{aligned}
\int_{\underline{r}}^{\overline{r}} F^B(r)\, r dF^A(r) &= \int_{\underline{r}}^{\overline{r}-} F^B(r)\, r dF^A(r) + F^B(\overline{r})\, \overline{r}\left[1 - F^A\left(\overline{r}^-\right)\right] \\
\underbrace{=}_{F^A = \frac{\alpha^B}{\alpha^A} F^B,\, F^B\left(\overline{r}^-\right) = F^B(\overline{r})} & \int_{\underline{r}}^{\overline{r}} F^A(r)\, r dF^B(r) + F^B(\overline{r})\, \overline{r}\left(1 - \frac{\alpha^B}{\alpha^A} F^B(\overline{r})\right)
\end{aligned}$$

Plug these terms into Eq. (43) and (44), and we have

$$\mathbb{E}\left[r^A\middle|r^A<r^B\leq\infty\right]=\frac{\mathbb{P}\left(g^A=H\right)\int_{\underline{r}}^{\overline{r}}rdF^A\left(r\right)-p_{HH}\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)rdF^A\left(r\right)}{p_{HH}\left[1-F^B\left(\overline{r}\right)+\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{HL}}$$
$$=\overline{r}-\frac{\mathbb{P}\left(g^A=H\right)\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr+p_{HH}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)-p_{HH}\overline{r}\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2}{p_{HH}\left[1-F^B\left(\overline{r}\right)+\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{HL}};$$

for Bank B,

$$\mathbb{E}\left[r^B\middle|r^B<r^A\leq\infty\right]=\frac{\mathbb{P}\left(g^B=H\right)\int_{\underline{r}}^{\overline{r}}rdF^B\left(r\right)-p_{HH}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)}{p_{HH}\left[F^B\left(\overline{r}\right)-\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{LH}F^B\left(\overline{r}\right)}$$
$$=\overline{r}-\frac{\mathbb{P}\left(g^B=H\right)\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr+p_{HH}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)-p_{HH}\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\overline{r}}{p_{HH}\left[F^B\left(\overline{r}\right)-\frac{\alpha^B}{2\alpha^A}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{LH}F^B\left(\overline{r}\right)}.$$

Therefore, $\mathbb{E}\left[r^A\middle|r^A<r^B\leq\infty\right]\geq\mathbb{E}\left[r^B\middle|r^B<r^A\leq\infty\right]$ is equivalent to the stated inequality. ☐

Lemma 7. *In the case of* $q>\frac{1}{1+\overline{r}}$*, when* $\alpha^B\uparrow\alpha^A$*, there exists a threshold* $\hat{\alpha}\left(\alpha^A\right)<\alpha^A$ *so that when* $\alpha^B>\hat{\alpha}\left(\alpha^A\right)$ *we have* $F^B\left(\overline{r}\right)=1$*.*

Proof. Let $\alpha^B=\alpha^A-\epsilon$. Bank B's profit could be pinned down by setting $r=\overline{r}^-$,

$$\begin{aligned}\pi^B=&p_{HH}\left[1-F^A\left(\overline{r}^-\right)\right]\left[\mu_{HH}\left(\overline{r}+1\right)-1\right]+p_{LH}\left[\mu_{LH}\left(\overline{r}+1\right)-1\right]\\ \underbrace{\geq}_{F^A\left(\overline{r}^-\right)\leq1}&\ p_{LH}\left(\mu_{LH}\left(\overline{r}+1\right)-1\right)\\ \underbrace{=}_{\alpha^B=\alpha^A-\epsilon}&\ q\left(1-\alpha^A\right)\left(\alpha^A-\epsilon\right)\overline{r}-\left(1-q\right)\alpha^A\left(1-\left(\alpha^A-\epsilon\right)\right)\\ =&\left(1-\alpha^A\right)\alpha^A\left[q\overline{r}-\left(1-q\right)\right]-\epsilon\left[q\left(1-\alpha^A\right)\overline{r}+\left(1-q\right)\alpha^A\right].\end{aligned}$$

Hence, when $\epsilon<\frac{\left(1-\alpha^A\right)\alpha^A\left[q\overline{r}-\left(1-q\right)\right]}{q\left(1-\alpha^A\right)\overline{r}+\left(1-q\right)\alpha^A}$, or equivalently, when

$$\alpha^B>\hat{\alpha}\left(\alpha^A\right)=\alpha^A-\frac{\left(1-\alpha^A\right)\alpha^A\left[q\overline{r}-\left(1-q\right)\right]}{q\left(1-\alpha^A\right)\overline{r}+\left(1-q\right)\alpha^A},$$

we have $\pi^B>0$ and $F^B\left(\overline{r}\right)=1$. ☐

Proof of Proposition 2 Part 2

Proof. There are two cases depending on whether $q<\frac{1}{1+\overline{r}}$, i.e., whether the project has a negative NPV at prior.

The first case of $q<\frac{1}{1+\overline{r}}$ is easier. When $\alpha^B\uparrow\alpha^A$ and $\alpha^A-\alpha^B=o\left(q-\frac{1}{1+\overline{r}}\right)$, Bank B's signal distributions and strategies approach that of Bank A except at $r=\overline{r}$ (Lemma 5):

$$F^B\left(r\right)\uparrow F^A\left(r\right)\quad\text{for any}\quad r\in\left[\underline{r},\overline{r}\right),\quad\text{and}\quad F^B\left(\overline{r}\right)<1=F^A\left(\overline{r}\right).$$

Then from Lemma 6,

$$\frac{\overline{r}-\mathbb{E}\left[r^A\middle| r^A<r^B\le\infty\right]}{\overline{r}-\mathbb{E}\left[r^B\middle| r^B<r^A\le\infty\right]} = \frac{p_{HH}\left[F^B\left(\overline{r}\right)-\frac{1}{2}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{LH}F^B\left(\overline{r}\right)}{p_{HH}\left[1-F^B\left(\overline{r}\right)+\frac{1}{2}\left(F^B\left(\overline{r}\right)\right)^2\right]+p_{HL}}$$
$$\underbrace{\le}_{\text{RHS set } F^B(\overline{r})=1} \frac{\frac{1}{2}p_{HH}+p_{LH}}{\frac{1}{2}p_{HH}+p_{HL}}=1, \tag{45}$$

where the last inequality holds because the ratio is increasing in $F^B\left(\overline{r}\right)$. ($F^B\left(\overline{r}\right)-\frac{1}{2}\left(F^B(\overline{r})\right)^2$ in both the numerator and denominator is monotone increasing when $F^B(\overline{r})\in(0,1]$.) Hence, $\mathbb{E}\left[r^A\middle| r^A<r^B\le\infty\right]\ge\mathbb{E}\left[r^B\middle| r^B<r^A\le\infty\right]$ always holds in this case.

Now consider the second case $q\ge\frac{1}{1+\overline{r}}$. When $\alpha^B\to\alpha^A$, since $\mathbb{E}\left[r^A\middle| r^A<r^B\le\infty\right]$ decreases while $\mathbb{E}\left[r^B\middle| r^B<r^A\le\infty\right]$ increases in $F^B\left(\overline{r}\right)$, it is sufficient to show that the equivalent inequality in Lemma 6 holds under $F^B\left(\overline{r}\right)=1$, i.e.,

$$\frac{\mathbb{P}\left(g^A=H\right)\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr+p_{HH}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)-p_{HH}\overline{r}\frac{\alpha^B}{2\alpha^A}}{p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}}$$
$$\le\frac{\mathbb{P}\left(g^B=H\right)\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr+p_{HH}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)-p_{HH}\frac{\alpha^B}{2\alpha^A}\overline{r}}{p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}}, \tag{46}$$

where both the LHS and RHS are positive. When $q>\frac{1}{1+\overline{r}}$, recall that Lemma 7 shows $F^B\left(\overline{r}\right)=1$ as $\alpha^B\to\alpha^A$ under $q>\frac{1}{1+\overline{r}}$ and so the inequality is also necessary.

Denote by $N\triangleq\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr>0$, and $M\triangleq\overline{r}\frac{\alpha^B}{2\alpha^A}-\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)$. $M>0$ because

$$\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)<\overline{r}\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)dF^B\left(r\right)=\overline{r}\int_{\underline{r}}^{\overline{r}}\frac{\alpha^B}{\alpha^A}F^B\left(r\right)dF^B\left(r\right)=\overline{r}\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}d\left(\frac{F^B\left(r\right)^2}{2}\right)=\overline{r}\frac{\alpha^B}{2\alpha^A}.$$

Collect terms in the key inequality (46), we have

$$\left\{\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}\right]\left(p_{HH}+p_{HL}\right)\frac{\alpha^B}{\alpha^A}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\left(p_{HH}+p_{LH}\right)\right\}N$$
$$\le p_{HH}\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\right]M \tag{47}$$

Let $\alpha^B=\alpha^A-\epsilon$ and calculate the coefficients. Note that as $\alpha^B=\alpha^A-\epsilon$, we have $p_{HL}-p_{LH}=(2q-1)\,\epsilon$.[26] The coefficient on the LHS of (47):

$$\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}\right]\left(p_{HH}+p_{HL}\right)\frac{\alpha^B}{\alpha^A}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\left(p_{HH}+p_{LH}\right)$$
$$=\left(\frac{p_{HH}}{2}+\frac{\epsilon}{2\alpha^A}p_{HH}+p_{LH}\right)\left(p_{HH}+p_{HL}\right)\left(1-\frac{\epsilon}{\alpha^A}\right)-\left(\frac{p_{HH}}{2}-\frac{\epsilon}{2\alpha^A}p_{HH}+p_{HL}\right)\left(p_{HH}+p_{LH}\right)$$
$$=-\frac{p_{HH}}{2}\left(2q-1\right)\epsilon+\frac{\epsilon}{2\alpha^A}p_{HH}^2-\frac{\epsilon}{2\alpha^A}p_{LH}p_{HH}-\frac{\epsilon}{\alpha^A}p_{LH}p_{HL}$$

[26] We have $p_{HL}=q\alpha^A\left(1-\alpha^B\right)+(1-q)\,\alpha^B\left(1-\alpha^A\right)$ and $p_{LH}=q\left(1-\alpha^A\right)\alpha^B+(1-q)\,\alpha^A\left(1-\alpha^B\right)$ and then therefore $p_{HL}-p_{LH}=q\left(\alpha^A-\alpha^B\right)+(1-q)\left(\alpha^B-\alpha^A\right)=(2q-1)\,\epsilon$.

The coefficient on the RHS of (47):

$$p_{HH}\left[p_{HH}\left(1-\frac{\alpha^B}{2\alpha^A}\right)+p_{LH}-\left(p_{HH}\frac{\alpha^B}{2\alpha^A}+p_{HL}\right)\right]=\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}\left(p_{HL}-p_{LH}\right)$$
$$=\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}\left(2q-1\right)\epsilon.$$

Plug the coefficients back into the inequality (47), so we need to show that

$$0\leq\left\{\frac{\epsilon}{\alpha^A}p_{HH}^2-p_{HH}\left(2q-1\right)\epsilon\right\}M-\left\{-\frac{p_{HH}}{2}\left(2q-1\right)\epsilon+\frac{\epsilon}{2\alpha^A}p_{HH}^2-\frac{\epsilon}{2\alpha^A}p_{LH}p_{HH}-\frac{\epsilon}{\alpha^A}p_{LH}p_{HL}\right\}N$$
$$=\left[\left(2q-1\right)-\frac{p_{HH}}{\alpha}\right]\frac{p_{HH}\left(N-2M\right)}{2}\epsilon+\left(\frac{1}{2}p_{LH}p_{HH}+p_{LH}p_{HL}\right)\frac{N}{\alpha}\epsilon.$$

Note that

$$N-2M=\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr-2\left(\overline{r}\frac{\alpha^B}{2\alpha^A}-\int_{\underline{r}}^{\overline{r}}F^A\left(r\right)rdF^B\left(r\right)\right)$$
$$=\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr-2\left(\overline{r}\frac{\alpha^B}{2\alpha^A}-\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)rdF^B\left(r\right)\right)$$
$$=\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr-2\left(\overline{r}\frac{\alpha^B}{2\alpha^A}-\frac{\alpha^B}{2\alpha^A}\overline{r}+\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}\frac{\left(F^B\left(r\right)\right)^2}{2}dr\right)$$
$$=\int_{\underline{r}}^{\overline{r}}F^B\left(r\right)dr-\frac{\alpha^B}{\alpha^A}\int_{\underline{r}}^{\overline{r}}\left(F^B\left(r\right)\right)^2dr>0.$$

Therefore, one sufficient condition is

$$2q-1\geq\frac{p_{HH}}{\alpha}=\frac{q\alpha^2+\left(1-q\right)\left(1-\alpha\right)^2}{\alpha}.$$

Collecting terms, the condition above requires $q\geq 1-\alpha+\alpha^2$. Since $1-\alpha+\alpha^2$ increases in α for $\alpha\in\left(\frac{1}{2},1\right)$, this imposes a simple condition that prior needs to be sufficiently good and information technology α cannot be too high. □

Calibration

Bad-news information structure. For the bad news information structure we need to calibrate the parameters $\left\{q,\alpha^A,\alpha^B\right\}$. To do so we use our Y14Q.H1 data for stress-tests banks, and take the NPL rates of specialized and non-specialized (stress-tested) banks (see Section B for more details). The two NPL rates are 3% (specialized) and 4% (non-specialized) as reported in Table 3. We also take the empirical loan approval rate of non-specialized banks as 70%, which lies in the middle of the range estimated in Yates (2020).

The theoretical counterparts for these three empirical moments are:

$$3\% = \mathbb{P}\left(\theta = 0 \left| r^A < r^B < \infty \right.\right) = \frac{\left(1-\alpha^A\right)\left(1-\alpha^B\right)\left\{\frac{1}{2}+\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\}+\left(1-\alpha^A\right)\alpha^B}{\left[\frac{\boldsymbol{q}}{\boldsymbol{1-q}}+\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]\left\{\frac{1}{2}+\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\}+\left(1-\alpha^A\right)\alpha^B},$$

$$4\% = \mathbb{P}\left(\theta = 0 \left| r^B < r^A < \infty \right.\right) = \frac{\left(1-\alpha^A\right)\left(1-\alpha^B\right)\left\{\frac{1}{2}-\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\}+\left(1-\alpha^B\right)\alpha^A F^B\left(\overline{r}\right)}{\left[\frac{q}{1-q}+\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]\left\{\frac{1}{2}-\frac{\left[1-F^B(\overline{r})\right]^2}{2}\right\}+\left(1-\alpha^B\right)\alpha^A F^B\left(\overline{r}\right)},$$

$$0.7 = \mathbb{P}\left(g^B = H\right) = q + (1-q)\left(1-\alpha^B\right),$$

where $F^B\left(\overline{r}\right) = \frac{\frac{\overline{r}}{\underline{r}}-1}{\frac{\overline{r}}{\underline{r}}-1+\alpha^A}$ and $\underline{r} = \frac{(1-q)\left(1-\alpha^B\right)}{q}$. Here, since we only observe the average loan approval rate in the banking sector, we calculate the approval rate for non-specialized bank, as empirically most of banks are non-specialized (the number of specialized banks reported in Table 4 is small relative to the total number of banks which is about 40 in our data).

We have seen from Part 1 of Proposition 2 that a negative interest rate wedge is more likely to arise for a higher $\overline{r}$. At the usury maximum interest rate cap 36%, the parameters $\left\{q, \alpha^A, \alpha^B\right\}$ that match the above three moments are $q = 0.6846$, $\alpha^A = 0.9655$ and $\alpha^B = 0.9513$. But even under this usury rate $\overline{r} = 36\%$, the interest rate wedge is positive (about 0.3%). In fact, we can calculate the implied threshold of $\hat{\overline{r}}$ (given $q = 0.6846$, $\alpha^A = 0.9655$ and $\alpha^B = 0.9513$) for a negative interest rate wedge, which takes a value of 395%—this is more than ten times of the usury rate 36%!

Symmetric information structure. For canonical models with a symmetric information structure, we gauge q and α from the limiting case where Bank B's information technology α^B approaches that of Bank A, i.e., $\alpha^B \to \alpha^A = \alpha$. We rely on two empirical moments for U.S. banks. First, we set the non-performing loan (NPL) rate to 4% to match the average NPL rate for stress-tests banks taken from Y14Q.H1 data (see Section B and in Table 3 for more details). Second, for robustness, we set the range for loan approval rates for business C&I loans from 55% (small lenders) to 80% (large lenders), following Yates (2020).

Depending on the primitives, Bank B may either make zero or positive profit in the unique equilibrium, which we call zero-weak or positive-weak analogous to our main equilibrium characterization with multi-dimensional information. Recall that at the beginning of the proof of Proposition 2, i.e., the discussion around condition (45), we have shown that negative interest rate wedge $\Delta r < 0$ fails in the zero-weak case where Bank B makes zero profit). Therefore we only need to consider the positive-weak case. In this case, we have $q \geq \frac{1}{1+\overline{r}}$ and lenders are symmetric: upon receiving a signal H each lender offers an interest rate drawn from the same distribution and wins with equal probabilities. Therefore, we can write the NPL ratio and approval rate of a bank, in this case Bank A, as

$$4\% = \frac{\mathbb{P}\left(\theta = 0 \middle| r^A < r^B < \infty\right)}{\mathbb{P}\left(r^A < r^B < \infty\right)} = \frac{(1-q)\left[\frac{(1-\alpha)^2}{2}+\alpha(1-\alpha)\right]}{(1-q)\left[\frac{(1-\alpha)^2}{2}+\alpha(1-\alpha)\right]+q\left[\frac{\alpha^2}{2}+\alpha(1-\alpha)\right]},$$

$$y = \mathbb{P}\left(g^A = H\right) = q\alpha + (1-q)(1-\alpha), \text{for } y \in [0.55, 0.80].$$

This system allows us to solve for the pair (q, α) for any value of y. For instance, when $y = 0.7$ one can solve for $q = 0.9217$ and $\alpha = 0.7371$, which satisfies the proposed sufficient condition $q \geq 1 - \alpha + \alpha^2$. The same conclusion holds for $y = 0.55$, so that $q = 0.9539$ and $\alpha = 0.5551$; or $y = 0.8$ so that $q = 0.8776$ and $\alpha = 0.8921$. (With these parameters, $q \geq \frac{1}{1+\overline{r}}$ always holds for $\overline{r} = 36\%$.)

A.4 Proof of Proposition 3

Proof. Based on the credit competition equilibrium in Proposition 1, the expected rates of a lender's issued loan are:

$$\mathbb{E}\left[r^A \middle| r^A < r^B \le \infty\right] = \frac{\underbrace{p_{HH}}_{g^A=g^B=H} \int_x^1 \underbrace{\left[1 - F^B\left(r^A(t)^-\right)\right]}_{A \text{ wins}} r^A(t)\,\phi(t)\,dt + \underbrace{p_{HL}}_{g^A=L, g^B=L} \int_x^1 r^A(t)\,\phi(t)\,dt}{p_{HH}\int_x^1 \left[1 - F^B\left(r^A(t)^-\right)\right]\phi(t)\,dt + p_{HL}\int_x^1 \phi(t)\,dt},$$

$$\mathbb{E}\left[r^B \middle| r^B < r^A \le \infty\right] = \frac{\underbrace{p_{HH}}_{g^A=g^B=H} \int_{\hat{s}}^1 \underbrace{\Phi(t)}_{B \text{ wins}} r(t)\,d\left[-F^B(r(t))\right] + \underbrace{p_{LH}}_{g^A=L, g^B=H} \int_x^1 r(t)\,dF^B(r(t))}{p_{HH}\int_{\hat{s}}^1 \Phi(t)\,d\left[-F^B(r(t))\right] + p_{LH}F^B(\bar{r})}.$$

In positive weak equilibrium, $F^B(r(s))$ has a point mass of size $1 - F^B\left(\bar{r}^-\right)$ at $\bar{r}$ or $r^A(\hat{s})$.

In this proposition, we impose the following conditions a) general signals are degenerate with $q_g = 1$ and b) $\bar{r} \to \infty$. (The logic for $\alpha_u = \alpha_d = 0.5$ so that lenders ignore the general signals are the same.) Then

$$\mathbb{E}\left[r^A + 1 \middle| r^A < r^B \le \infty\right] = \frac{\int_0^1 \left[1 - F^B\left(r^A(t)^-\right)\right] r^A(t)\,\phi(t)\,dt}{\int_0^1 \left[1 - F^B\left(r^A(t)^-\right)\right]\phi(t)\,dt} = \frac{\int_0^1 \Phi(t)\,\phi(t)\,dt}{\int_0^1 \left[\int_0^t \nu\phi(\nu)\,dt\right]\phi(t)\,dt},$$

$$\mathbb{E}\left[r^B + 1 \middle| r^B < r^A \le \infty\right] = \frac{\int_0^1 \Phi(t)\,r(t)\,d\left[-F^B(r(t))\right]}{\int_0^1 \Phi(t)\,d\left[-F^B(r(t))\right]} = \frac{\int_0^1 \Phi(t)\left[\frac{t\Phi(t)}{\int_0^t \nu\phi(\nu)d\nu}\right]\phi(t)\,dt}{\int_0^1 \Phi(t)\,t\phi(t)\,dt},$$

where the first equality of both variables uses condition a) degenerate signals and $x = \hat{s} = 0$ which follows from condition b), and the second equality uses equilibrium strategy $r^A(t) = \frac{\Phi(s)}{\int_0^s t\phi(t)dt}$ and $1 - F^B\left(r^A(t)^-\right) = \frac{\int_0^t \nu\phi(\nu)dt}{q_s}$.

Additionally, c) the specialized signal distribution is $\phi(s) = 1 + \epsilon\left[2 \cdot \mathbf{1}_{s \le 0.5} - 1\right]$. Then

$$\mathbb{E}\left[r^A + 1 \middle| r^A < r^B \le \infty\right] = 2 \cdot \frac{\frac{1}{8}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{2} + \frac{3}{8}(1-\epsilon)^2}{\frac{1}{24}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{4} + \frac{7}{24}(1-\epsilon)^2},$$

$$\mathbb{E}\left[r^B + 1 \middle| r^B < r^A \le \infty\right] = 2 \cdot \frac{\frac{1}{8}(1+\epsilon)^2 + \frac{\epsilon(1-\epsilon)}{2} + \frac{3}{8}(1-\epsilon)^2 + \epsilon^2(1-\epsilon)\int_{0.5}^1 \frac{\left(t-\frac{1}{2}\right)}{\frac{\epsilon}{2} + (1-\epsilon)t^2}dt + \epsilon(1-\epsilon)^2\int_{0.5}^1 \frac{t\left(t-\frac{1}{2}\right)}{\frac{\epsilon}{2} + (1-\epsilon)t^2}dt}{\frac{1}{24}(1+\epsilon)^2 + \frac{3\epsilon(1-\epsilon)}{8} + \frac{7}{24}(1-\epsilon)^2}.$$

Note that when $\epsilon = 0$, $\Delta r = 0$. When $\epsilon \to 0$, we have (ignoring higher order terms of ϵ)

$$\frac{\partial \Delta r}{\partial \epsilon} = \lim_{\epsilon \to 0} \frac{\Delta r(\epsilon)}{\epsilon} = \frac{1}{\epsilon}\left(\frac{1}{\frac{1}{3} - \frac{1}{4}\epsilon} - \frac{1 + \epsilon - \epsilon \ln 2}{\frac{1}{3} - \frac{1}{8}\epsilon}\right) = 3\ln 2 - \frac{15}{8} > 0.$$

Hence, when $\epsilon > 0$ ($\epsilon < 0$), i.e., $\phi(s)$ tilts toward less (more) favorable realizations, we have $\Delta r > 0$ ($\Delta r < 0$). □

A.5 General Information Structure

In this extension, we focus on the well-behaved structure (i.e., smooth distribution of interest rates over $[\underline{r}, \bar{r})$ and decreasing $r^A(z)$) and show that the lender strategies in Proposition 4 correspond to an equilibrium.

We impose the following primitive conditions under which the general signal is decisive.

Assumption 2. *i) Bank A rejects the borrower upon an L general signal, regardless of any specialized signal* *z:*

$$\mu_{L\cdot}(\overline{z})(\overline{r}+1)-1<0. \tag{48}$$

ii) Bank B is willing to participate if and only if its general signal $g^B = H$:

$$\int_{\underline{z}}^{\overline{z}} p_{\cdot H}(t)\left[\mu_{\cdot H}(t)(\overline{r}+1)-1\right]dt>0. \tag{49}$$

Proof of Proposition 4

Proof. Similar as the derivation in the baseline model, we first take π^B as given to characterize lender strategy, and then solve for π^B.

Bank A's strategy

In the region of $z \in (\hat{z}, 1]$ that corresponds to $r^A(z) \in [\underline{r}, \overline{r})$, $r^A(\cdot)$ is strictly decreasing so the inverse function $z^A(\cdot) \equiv r^{A(-1)}(\cdot)$ is properly defined. Bank B's lending profit when quoting $r \in [\underline{r}, \overline{r})$ is

$$\begin{aligned}\pi^B(r) &= \underbrace{\overline{p}_{HH}}_{g^A=H}\cdot\underbrace{\int_{\underline{z}}^{z^A(r)}}_{B\text{ wins}}\left[\underbrace{\mu_{HH}(t)}_{\text{repay}}(1+r)-1\right]\phi_z(t|\,HH)\,dt+\underbrace{\overline{p}_{LH}}_{g^A=L}\left[\underbrace{\overline{\mu}_{LH}}_{\text{repay}}(1+r)-1\right]\\ &=(1+r)\left[\int_{\underline{z}}^{z^A(r)}p_{HH}(t)\,\mu_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}\right]-\int_{\underline{z}}^{z^A(r)}p_{HH}(t)\,dt-\overline{p}_{LH}\end{aligned} \tag{50}$$

Bank A's equilibrium strategy $r^A(z)$ for $z \in [\hat{z}, 1]$ is such that Bank B is indifferent across $r \in [\underline{r}, \overline{r})$. Hence,

$$r^A(z)=\frac{\overbrace{\pi^B+\int_{\underline{z}}^{z}p_{HH}(t)\,dt+\overline{p}_{LH}}^{B\text{'s lending amount}}}{\underbrace{\int_{\underline{z}}^{z}p_{HH}(t)\cdot\mu_{HH}(t)\,dt+\overline{p}_{LH}\overline{\mu}_{LH}}_{B\text{'s customers who repay}}}-1,\quad\text{where}\quad \hat{z}\le s\le\overline{z}. \tag{51}$$

Note that this pins down $\underline{r} = (r^A)^{-1}(\overline{z})$ which is a function of π^B.

In addition, $r^A(z) = \overline{r}$ for $z \in [z_x, \hat{z})$ and Bank A rejects the borrower when $z \in [\underline{z}, z_x)$, where z_x satisfies

$$\pi^A\left(r^A(z_x)=\overline{r}\middle|\,z_x\right)=0.$$

This completes the proof of Bank A's strategy in Proposition 4.

Bank B's strategy

Bank A's offered interest rate $r^A(z)$ upon $z \in [\hat{z}, \overline{z}]$ maximizes

$$\pi^A\left(r^A(z)\middle|\,z\right)=\underbrace{p_{HH}(z)}_{g^B=H}\underbrace{\left[1-F^B(r)\right]}_{A\text{ wins}}\left[\underbrace{\mu_{HH}(z)}_{\text{repay}}(1+r)-1\right]+\underbrace{p_{HL}(z)}_{g^B=L}\left[\underbrace{\mu_{HL}(z)}_{\text{repay}}(1+r)-1\right]$$

The FOC with respect to r is

$$\underbrace{\left[-\frac{d\left[F^B(r)\right]}{dr}\right]}_{\Delta\text{winning prob}}\underbrace{p_{HH}(z)\left[\mu_{HH}(z)(1+r)-1\right]}_{\text{profit upon winning}}+\underbrace{p_{HH}(z)\left[1-F^B(r)\right]\mu_{HH}(z)+p_{HL}(z)\,\mu_{HL}(z)}_{\text{existing customer}}=0.$$

Bank A's optimal strategy $r^A(z)$ satisfies this first-order condition,

$$\begin{aligned} 0 = & -\frac{d\left[F^B\left(r^A(z)\right)\right]}{dr} p_{HH}(z)\left[\mu_{HH}(z)\left(1+r^A(z)\right)-1\right] \\ & + p_{HH}(z)\left[1-F^B\left(r^A(z)\right)\right]\mu_{HH}(z) + p_{HL}(z)\,\mu_{HL}(z). \end{aligned} \tag{52}$$

From Eq. (51) about $r^A(z)$, we derive the following key equation by taking derivatives w.r.t. z,

$$\underbrace{\frac{dr^A(z)}{dz}\left[\int_{\underline{z}}^{z} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]}_{\text{B: }\uparrow\text{marginal customer return}} + \underbrace{p_{HH}(z)\left[\left(r^A(z)+1\right)\mu_{HH}(z)-1\right]}_{\text{B: }\uparrow\text{existing customer revenue}} = 0.$$

Plug this equation into the FOC (52), and we have

$$-\frac{d\left[F^B\left(r^A(z)\right)\right]}{dz}\left[\int_{\underline{z}}^{z} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right] = p_{HH}(z)\left[1-F^B(r)\right]\mu_{HH}(z) + p_{HL}(z)\,\mu_{HL}(z),$$

which is equivalent to

$$\frac{d}{dz}\left\{\frac{1-F^B\left(r^A(z)\right)}{\int_{\underline{z}}^{z}\mu_{HH}(t)\,p_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right\} = \frac{p_{HL}(z)\,\mu_{HL}(z)}{\left[\int_{\underline{z}}^{z} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2}. \tag{53}$$

Since signals are independent conditional on the state being $\theta = 1$, the right-hand-side equals

$$\begin{aligned} & \frac{q\mathbb{P}(HL|\,\theta=1)\,\phi_z(z|\,\theta=1)}{\left[\int_{\underline{z}}^{z} q\mathbb{P}(HH|\,\theta=1)\,\phi_z(t|\,\theta=1)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]^2} \\ = & -\frac{\mathbb{P}\left(g^B=L\middle|\,\theta=1\right)}{\mathbb{P}(g^B=H|\,\theta=1)}\frac{d}{dz}\left[\frac{1}{\int_{\underline{z}}^{z} q\mathbb{P}(HH|\,\theta=1)\,\phi_z(t|\,\theta=1)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right]. \end{aligned}$$

Then the solution $F^B\left(r^A(z)\right)$ to the ODE (53) satisfies

$$\frac{1-F^B\left(r^A(z)\right)}{\int_{\underline{z}}^{z}\mu_{HH}(t)\,p_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}} = -\frac{\mathbb{P}\left(g^B=L\middle|\,\theta=1\right)}{\mathbb{P}(g^B=H|\,\theta=1)}\left[\frac{1}{\int_{\underline{z}}^{z}\mu_{HH}(t)\,p_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right] + Const.$$

Using the boundary condition $F^B\left(r^A(\overline{z}) = \underline{r}\right) = 0$, we solve for the constant

$$Const = \frac{1}{\mathbb{P}(\theta=1)}\frac{1}{\mathbb{P}(g^B=H|\,\theta=1)^2}.$$

Therefore,

$$\begin{aligned} F^B(r) = & \frac{1}{\mathbb{P}(g^B=H|\,\theta=1)} - \frac{\int_{\underline{z}}^{z^A(r)}\mu_{HH}(t)\,p_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}{\mathbb{P}(\theta=1)\,\mathbb{P}(g^B=H|\,\theta=1)^2} \\ = & \frac{1}{\mathbb{P}(g^B=H|\,\theta=1)} - \frac{\mathbb{P}(\theta=1)\,\mathbb{P}(HH|\,\theta=1)\int_{\underline{z}}^{z^A(r)}\phi_z(t|\,\theta=1)\,dt + \mathbb{P}(\theta=1)\,\mathbb{P}(LH|\,\theta=1)}{\mathbb{P}(\theta=1)\,\mathbb{P}(g^B=H|\,\theta=1)^2} \\ = & \frac{\mathbb{P}\left(g^A=H\middle|\,\theta=1\right)}{\mathbb{P}(g^B=H|\,\theta=1)}\left[1-\int_{\underline{z}}^{z^A(r)}\phi_z(t|\,\theta=1)\,dt\right]. \end{aligned}$$

Bank B's profit π^B

Now we are left with one unknown variable π^B in Eq. (51). Similar to the baseline model, the equilibrium could be positive-weak or zero-weak, depending on who—Bank A receiving threshold specialized signal z_A^{be} and quoting $\overline{r}$ or Bank B—breaks even first in competition. We define z_A^{be} and z_B^{be} as

$$\begin{aligned} 0 = \pi^A\left(\overline{r}\left|z_A^{be}\right.\right) =& p_{HH}\left(z_A^{be}\right)\frac{\mathbb{P}\left(g^A = H\middle|\theta=1\right)}{\mathbb{P}\left(g^B = H\middle|\theta=1\right)}\left[1-\int_{\underline{z}}^{z_A^{be}}\phi_z\left(t\middle|\theta=1\right)dt\right]\cdot\left[\mu_{HH}\left(z_A^{be}\right)(1+\overline{r})-1\right] \\ &+ p_{HL}\left(z_A^{be}\right)\left[\mu_{HL}\left(z_A^{be}\right)(1+\overline{r})-1\right], \\ 0 = \pi^B\left(\overline{r};z_B^{be}\right) =& \int_{\underline{z}}^{z_B^{be}} p_{HH}(t)\,\mu_{HH}(t)\,(1+\overline{r})\,dt - \int_{\underline{z}}^{z_B^{be}} p_{HH}(t)\,dt + \overline{p}_{HL}\left[\overline{\mu}_{HL}(1+\overline{r})-1\right]. \end{aligned}$$

Equilibrium π^B is then

$$\pi^B = \max\left\{\int_{\underline{z}}^{z_A^{be}} p_{HH}(t)\,\mu_{HH}(t)\,(1+\overline{r})\,dt - \int_{\underline{z}}^{z_A^{be}} p_{HH}(t)\,dt + \overline{p}_{HL}\left[\overline{\mu}_{HL}(1+\overline{r})-1\right], 0\right\}.$$

When $z_A^{be} > z_B^{be}$, equilibrium is positive weak with $\pi^B > 0$, and $\hat{z} = z_x = z_A^{be}$; when $z_A^{be} \leq z_B^{be}$, equilibrium is zero weak with $\pi^B = 0$, and $z_B^{be} = \hat{z} > z_x$. □

A.6 Information Acquisition

In this part, we first characterize lending profits and then provide a numerical illustration in which the specialization equilibrium arises.

Lending Profits

We characterize lending profits as a function of information acquisition, $\Pi_A\left(I_A^g, I_A^s, I_B^g, I_B^s\right)$ (we focus on Bank A due to symmetry.) We omit the case where there is an uninformed lender.

$\mathbf{I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0}$ **(Specialization).** This is the equilibrium that we focus on—each lender has a general information signal and only Bank A has a specialized signal s. Bank A's expected lending profit before signal realizations is thus

$$\Pi_A\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \int_x^1 \pi^A\left(r^A(s)\middle|\, s\right)\phi(s)\,ds,$$

where $\pi^A\left(r^A(s)\middle|\, s\right)$ is the profits for given signal realizations H and s and is given in Eq. (9). Using the equilibrium strategies in Proposition 1, we have

$$\pi^A\left(r^A(s)\middle|\, s\right) = p_{HH}\cdot\frac{\int_0^{\max\{s,\hat{s}\}}(s-t)\,\phi(t)\,dt}{q_s} + \left(\pi^B + p_{LH}\right)\cdot\frac{s}{q_s} - p_{HL}, \text{ for } s \geq x.$$

The expression shows that Bank A earns the information rent from the specialized signal. Bank A observes s, while Bank B may only negatively update the prior q_s when winning the competition that $s^A \leq s(r)$; this is reflected in the terms $\frac{s}{q_s}$ and $\frac{\int_0^{\min\{s,\hat{s}\}}(s-t)\phi(t)dt}{q_s}$.

In this case, Bank B's profit $\Pi_B\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \pi^B$ is given in Lemma 2. By symmetry, $\Pi_A\left(I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 1\right) = \Pi_B\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0\right) = \pi^B$.

$\mathbf{I_A^g = 0, I_A^s = 1, I_B^g = 1, I_B^s = 0}$ **(Asymmetric technology).** In this case, Bank A only collects specialized information while Bank B only collects general information in industry a. This case is nested in the previous case of specialization ($I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 0$), by reformulating Bank A to have an uninformative general signal, e.g.,

$$\mathbb{P}\left(g^A = H \middle| \theta_g = 1\right) = \mathbb{P}\left(g^A = H \middle| \theta_g = 0\right) = 1.$$

$\mathbf{I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 0}$ **(General information only).** In this case, both lenders only acquire general information, i.e., investing in IT and data processing that apply to both industries. The credit competition corresponds to Broecker (1990) with two lenders. Lenders are symmetric and the lending profit of, say Bank A, is

$$\Pi_A\left(I_A^g = 1, I_A^s = 0, I_B^g = 1, I_B^s = 0\right) = \max\left\{p_{HL}\left(\mu_{HL} q_s \overline{r} - 1\right), 0\right\}.$$

The "max" operator arises because either both lenders withdraw with positive probability (zero profits), or both lenders make profits and neither has a point mass at $\overline{r}$, i.e., $F^j\left(\overline{r}^-\right) = 1$.

$\mathbf{I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 1}$ **(Acquire all information).** In this symmetric case, each lender invests in both information technologies and receives both the general and specialized signals. We characterize the credit market equilibrium based on Riordan (1993) which considers the competition between two lenders each with a continuous private signal. Here, each lender additionally has a binary signal that represents the general information. Following the modeling of Riordan (1993), we work with the direct specialized signal z. Specifically, let z and Z denote the realization and the random variable of the specialized signal respectively, and let

$$\tilde{F}(z) \equiv \mathbb{P}\left(Z \le z \middle| \theta_s = 1\right), \ \tilde{G}(z) \equiv \mathbb{P}\left(Z \le z \middle| \theta_s = 0\right)$$

denote the CDFs of Z conditional on the underlying state θ_s, with the corresponding PDFs denoted by $\tilde{f}$ and $\tilde{g}$. Introduce $\mu(z) \equiv \mathbb{P}\left(\theta_s = g \middle| S\right)$ as the posterior belief, which is s in our baseline model.

A lender only bids when the general signal is H and the specialized signal $z \ge x$. Let $R(z) \equiv r(z) + 1$ denote the equilibrium gross rate quote. Given competitor's strategy $R(z)$, the lending profits from any R is then

$$\begin{aligned}\pi\left(R \middle| z\right) = {} & \left[p_{HH}\mu_{HH}\mu(z)\tilde{F}(t(R)) + p_{HL}\mu_{HL}\mu(z)\right] R \\ & - p_{HH}\left[(1-\mu(z))\tilde{G}(t(R)) + \mu(z)\tilde{F}(t(R))\right] - p_{HL},\end{aligned} \tag{54}$$

where $t(R)$ the signal such that the other bank offers R. The first order condition w.r.t. R is

$$\begin{aligned}\frac{\partial \pi\left(R(t) \middle| z\right)}{\partial R} = {} & \left[p_{HH}\mu_{HH}\mu(z)\tilde{F}(t) + p_{HL}\mu_{HL}\mu(z)\right] \\ & + \left\{p_{HH}\mu_{HH}\mu(z)\tilde{f}(t)R(t) - p_{HH}\left[(1-\mu(z))\tilde{g}(t) + \mu(z)\tilde{f}(t)\right]\right\}\frac{dt}{dR}.\end{aligned}$$

The equilibrium strategy satisfies

$$\left.\frac{\partial \pi\left(R(t) \middle| z\right)}{\partial t}\right|_{t=z} = 0.$$

By symmetry, we have

$$\frac{dt}{dR} = \frac{1}{R'(t)}.$$

These two conditions imply

$$p_{HH}\mu_{HH}\tilde{f}(z)R(z) + \left(p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL}\right)R'(z) = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}, \tag{55}$$

or equivalently,

$$\frac{d\left\{\left[p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL}\right]R(z)\right\}}{dz} = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}.$$

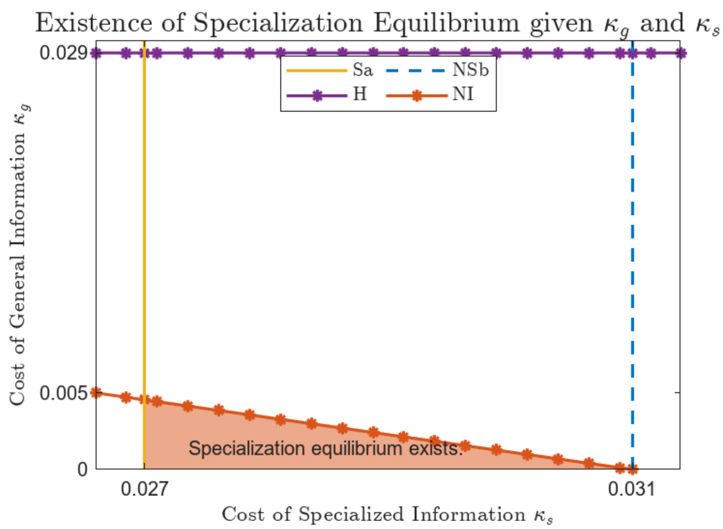

Figure 6: **Specialization Equilibrium.** This figure depicts the incentive compatibility constraints where Bank A does not want to deviate from the specialization equilibrium. Parameters: $\bar{r} = 0.36$, $\rho_h = 0$, $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$. Note τ captures the signal-to-noise ratio of Bank A's specialized information technology as $s = \mathbb{E}\left[\theta_s | \theta_s + \epsilon\right]$ and $\epsilon \sim \mathcal{N}\left(0, 1/\tau\right)$.

Integrating over z, we have

$$R\left(z\right) = \frac{\int_{\underline{z}}^{z} \frac{p_{HH}\left(1-\mu(t)\right)\tilde{g}(t)+p_{HH}\mu(t)\tilde{f}(t)}{\mu(t)} dt + constant}{p_{HH}\mu_{HH}\tilde{F}\left(z\right) + p_{HL}\mu_{HL}}. \quad (56)$$

The unknown constant is pinned down by the boundary condition $\pi\left(\bar{r} + 1 \,| x\right) = 0$: Upon the threshold signal x, a lender quotes the maximum interest rate $\bar{r} + 1$ and makes zero profit,

$$0 = \left[p_{HH}\mu_{HH}\mu\left(x\right)\tilde{F}\left(x\right) + p_{HL}\mu_{HL}\mu\left(x\right)\right]\left(\bar{r} + 1\right) - p_{HH}\left[\left(1 - \mu\left(x\right)\right)\tilde{G}\left(x\right) + \mu\left(x\right)\tilde{F}\left(x\right)\right] - p_{HL}. \quad (57)$$

Then a lender's lending profit is

$$\Pi_A\left(I_A^g = 1, I_A^s = 1, I_B^g = 1, I_B^s = 1\right) = \int_x^{\bar{z}} \pi\left(R\left(z\right) | \, z\right)\left[q_s\tilde{f}\left(z\right) + \left(1 - q_s\right)\tilde{g}\left(z\right)\right] dz,$$

where $R\left(z\right)$ is given by Eq. (56) and (57), profit $\pi\left(R\left(z\right), z\right)$ is given by Eq. (54).

Specialization Equilibrium

Figure 6 shows the region of information acquisition costs κ_h and κ_s to support the specialization equilibrium so that one of the banks endogenously becomes the specialized bank in one industry by acquiring both specialized and general information while the other is non-specialized by acquiring the general information only. In sum, we need κ_h to be sufficiently small while κ_s to lie in an intermediate range.

B Empirical Analyses

Data We use Y14Q-H.1 data that is collected by the Federal Reserve System as part of its stress-testing efforts, covering all C&I loans to which a stress-tested bank has committed more than 1 million USD (around 75% of all U.S. C&I lending). As such, the data covers 40 banks – in an unbalanced panel – between 2012 and 2023 and includes millions of loan-quarter observations.

Table 3: **Summary Statistics of Key Variables**

	N	Mean	SD	Specialized	Non-Specialized	Differential
Interest Rate	353,544	3.69	1.64	3.55	3.69	-0.13***
Non-Performing	353,544	0.04	0.19	0.03	0.04	-0.01***
Loan Amount	353,544	12.42	5.43	10.5	12.99	2.5***

Note: This table shows summary statistics for loans in our sample. We count each bank-loan combination only once, on the date when it is first observed in our data (this may be a different date from the loan's first origination date for a small subset of loans only as we censor our data and start in 2012, one year after collection began in 2011). Loan size is scaled by 1 million USD. The interest rate is the unadjusted cost of the loan, measured in percent."Non performing" is a dummy that takes the value of 1 if the loan ever falls in arrears, has negative maturity or is otherwise in default after the first observation in our sample. The mean values of each variable data are split by whether a loan is made by a specialized bank or not.

We focus on term loans and limit our sample to loans that are likely newly originated or new to the lender. We cut our data before 2012 to avoid accidentally labeling a loan as "newly originated," simply because of the point at which the data collection begins. We define a loan as new when it first appears in our data. We remove loans to financial or insurance entities. Our final sample covers 350,000 new term loans. Besides loan amount, we can track key loan data such as the interest rate paid by the borrower, the loan's purpose, and the performance of the loan while it remains in our sample, as we can see if it ever falls into arrears.

Summary statistics Key summary statistics for loans in our sample are outlined in Table 3. The average loan commitment in our sample is just over 12 million USD in size and the average loan interest rate is 3.7%. We define a loan as non-performing if it is ever 90+ days in arrears, ever has negative maturity (i.e. has not been repaid at maturity), or has outright defaulted. We then take a loan as "ever" non-performing if it becomes so at any point after origination. The percentage of non-performing loans is around 4% in our data, which is slightly higher than the average default rate given our wider definition.

Competitiveness of lending market. In Section 4.2 we also study the interaction between specialization and lending market competition, in order to rule out the alternative hypothesis that negative interest wedge is driven by competition among specialized lenders. There, we define an industry as "competitive" if two or more banks specialize in it.

Table 4 lists the number of banks that are specialized in industries in our data. There, we have obscured the exact industry definition in favor of stylized industry names in Table 4, though each represents a two-digit industry (with the omission of finance and insurance). As shown in Table 4, in our data the number of banks that are specialized in industries varies greatly. Some industries are home to no specialized banks, while other industries see nine banks that are specialized.

Table 4: **Number of Banks Specialized per Industry**

Industry	Number of Specialized Banks
A	0
B	1
C	3
D	0
E	2
F	2
G	4
H	3
I	7
J	0
K	2
L	0
M	3
N	9
O	2
P	1
Q	0
R	3
S	9
T	1
U	4
V	3
W	5

Note: We indicate the number of banks specialized in stylized industries. We define a bank as specialized if it is over-invested by 4% or more in an industry, relative to what would be expected from diversification (i.e. a bank that invests 14% of its C&I portfolio in an industry that accounts for 10% of all C&I lending would be specialized in that industry.) An industry is competitive if 2 or more banks are specialized in it.