The Value of Arbitrage

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Motivation

- Absence of Arbitrage $\Rightarrow$ Pillar of modern finance
- Active (empirical) literature documents violations of the Law-of-One-Price
  - CIP, Swap spreads, ADR's, etc.

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- These results beget the question:

What is the (social) value of closing an arbitrage gap?

- Alternatively: What are the costs associated with these violations?

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5. Measures of this bound for several scenarios (not today)
6. Extensions (not today)

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- Type A investors solve

$$\max_{q_0^A} u_A\left(c_0^A\right) + \beta^A u_A\left(c_1^A\right)$$

subject to

$$\begin{aligned} p^A q_0^A + c_0^A &= n_0^A + p^A s^A \\ c_1^A &= n_1^A + d_1 q_0^A \end{aligned}$$

- Type B investors solve the same problem
  - Potentially different preferences and endowments

Environment

- Arbitrageur sector solves

$$\max_{q_0^{A\alpha}, q_0^{B\alpha}} p_A q_0^{A\alpha} + p_B q_0^{B\alpha}$$

subject to

$$\beta d_1 \left(q_0^{A\alpha} + q_0^{B\alpha} \right) = 0$$

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- Assumption (w.l.o.g): $p^A < p^B$ in autarky

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- Differential methods
- **We abstract from the the friction that limits m**

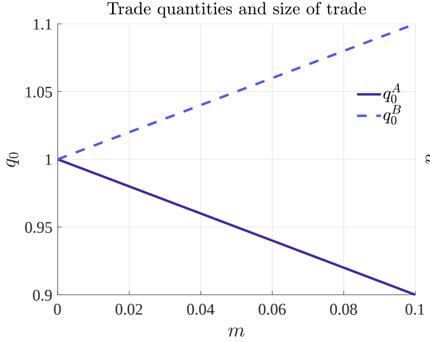
Arbitrage gap

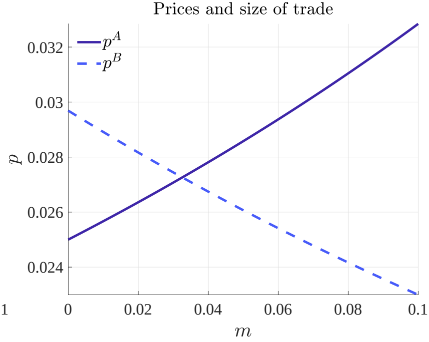
- *Arbitrage gap* definition:

$$\mathcal{G}_{BA}(m) := p^B(m) - p^A(m)$$

- Lemma 1: ***Arbitrage gap narrows with the amount arbitraged***

$$\mathcal{G}'_{BA}(m) = \frac{dp^B}{dm} - \frac{dp^A}{dm} < 0$$





Marginal Value of Arbitrage

Proposition 1: Marginal Value of Arbitrage

a) The marginal value of arbitrage is given by

$$\frac{\frac{dV^A}{dm}}{u'\left(c_0^A\right)} = \underbrace{\frac{dp^A}{dm}}_{>0} \underbrace{\left(s^A - q_0^A\right)}_{=m} > 0$$

$$\frac{\frac{dV^B}{dm}}{u'\left(c_0^B\right)} = \underbrace{\frac{dp^B}{dm}}_{<0} \underbrace{\left(s^B - q_0^B\right)}_{=-m} > 0$$

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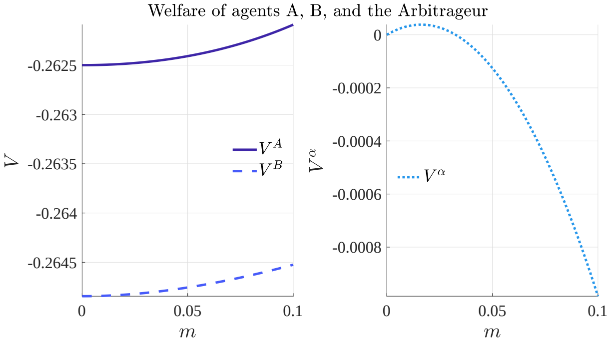
b) When aggregated, in date 0 dollars, (or if $m = 0$)

$$\frac{\frac{dV^A}{dm}}{u'\left(c_0^A\right)} + \frac{\frac{dV^B}{dm}}{u'\left(c_0^B\right)} + \frac{dV^\alpha}{dm} = p_B - p_A > 0$$

Marginal Value of Arbitrage

- Direct effect + distributive pecuniary externalities (cancel out)



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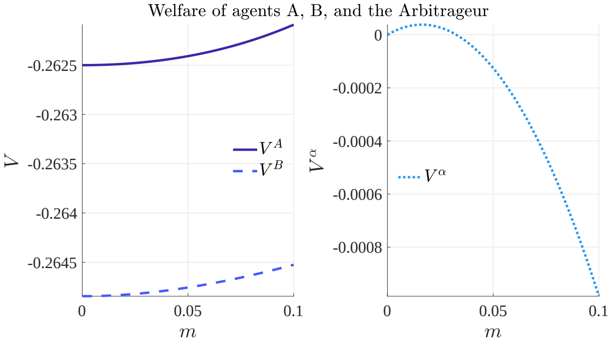

- **Distributional consequences:**
  - Investors in markets A and B benefit from increasing the arbitrage trade m (distributive effects)
  - Arbitrageur sector is initially better off, but there is a maximum
  - Arbitrage is Pareto improving (even with a monopolist arbitrageur)

Total Value of Arbitrage

Proposition 2: Total Value of Arbitrage + Exact Upper Bound

The total value of arbitrage is given by

$$W(m^*) - W(m_0) = \int_{m_0}^{m^*} W'(m)\, dm = \int_{m_0}^{m^*} \mathcal{G}_{BA}(m)\, dm \leq \mathcal{G}_{BA}(m_0)(m^* - m_0),$$

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- Exploit Lemma 1 to get inequality ($W(m)$ is concave)
- Can we find an upper bound computable only with information as of m_0? Yes, by approximating $m^* - m_0$

Total Value of Arbitrage

Proposition 3: Approximate Upper Bound

The total value of arbitrage can be approximated as

$$W(m^*) - W(m_0) \lesssim \frac{\left(\mathcal{G}_{BA}(m_0)\right)^2}{\frac{dp^A}{dm}(m_0) - \frac{dp^B}{dm}(m_0)}$$

- RHS is expressed as a function of m_0 (status quo)
- Total value of arbitrage is quadratic in the gap

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- RHS is expressed as a function of m_0 (status quo)
- Total value of arbitrage is quadratic in the gap
- $\frac{dp^A}{dm}$ and $\frac{dp^B}{dm}$ are measures of **price impact**
  - Quantity measures are needed
- If price impact is
  - convex $\Rightarrow$ Approximation is an upper bound
  - linear $\Rightarrow$ Approximation is exact

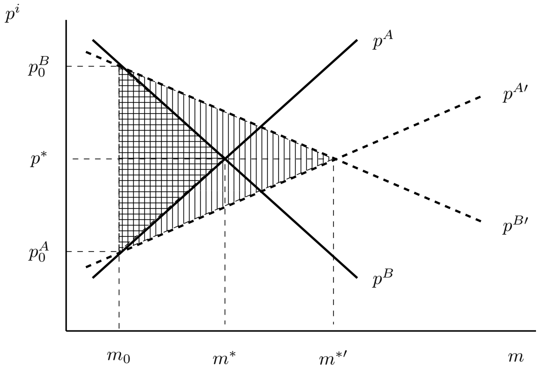
Liquidity and the value of arbitrage

- Liquid markets $\Rightarrow$ Large price impact

Proposition 4: Liquidity

For a given arbitrage gap $p^A - p^B$, the total social value of arbitrage is higher (lower) in more liquid (illiquid) markets

Illustration



- Shaded areas measure total value of arbitrage
- High price impact (steep curves) $\Rightarrow$ Small gains
- Low price impact (flat curves) $\Rightarrow$ Large gains

Additional Results

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  - Multiple assets
  - Multiple periods
  - Explicit microfoundations for limits to arbitrage
    - Constraints
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- Extensions
  - Multiple assets
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  - Explicit microfoundations for limits to arbitrage
    - Constraints
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- Measurement
  - Proposition 3 provides the basis for the empirical implementation

Summary

- Framework to compute the value of arbitrage
  - Stylized but easily generalizable framework
- Marginal and total measures
- Distributional impact
- Role of liquidity
- Really exciting to compute the estimates (coming very soon)